1.1. Definition

The subject **Theory of Machines** may be defined as that branch of Engineering-science, which deals with the study of relative motion between the various parts of a machine, and forces which act on them. The knowledge of this subject is very essential for an engineer in designing the various parts of a machine.

Note: A machine is a device which receives energy in some available form and utilises it to do some particular type of work.

1.2. Sub-divisions of Theory of Machines

The Theory of Machines may be sub-divided into the following four branches :

1. ***Kinematics.*** It is that branch of Theory of Machines which deals with the relative motion between the various parts of the machines.

2. ***Dynamics.*** It is that branch of Theory of Machines which deals with the forces and their effects, while acting upon the machine parts in motion.

3. ***Kinetics.*** It is that branch of Theory of Machines which deals with the inertia forces which arise from the combined effect of the mass and motion of the machine parts.

4. ***Statics.*** It is that branch of Theory of Machines which deals with the forces and their effects while the machine parts are at rest. The mass of the parts is assumed to be negligible.

Mechanical Engg.

↓

Engg. of mechanics

↓

study of motion

Study of motion without consideration of basic cause of motion, i.e. force

↓

Kinematics

$\vec{V} = \frac{d\vec{s}}{dt}$ m/s

$\vec{a} = \frac{d\vec{V}}{dt}$ m/s²

$\vec{j} = \frac{d\vec{a}}{dt}$ m/s³

(no term of mass)

study of motion with the consideration of basic cause of motion i.e. force

↓

Dynamics

NSL

$\vec{F}_{ext} = \frac{d}{dt}(m\vec{V})$

↓ mass

kinematic viscosity

$\nu = \frac{\mu}{\rho} = m^2/s$

Dynamic viscosity

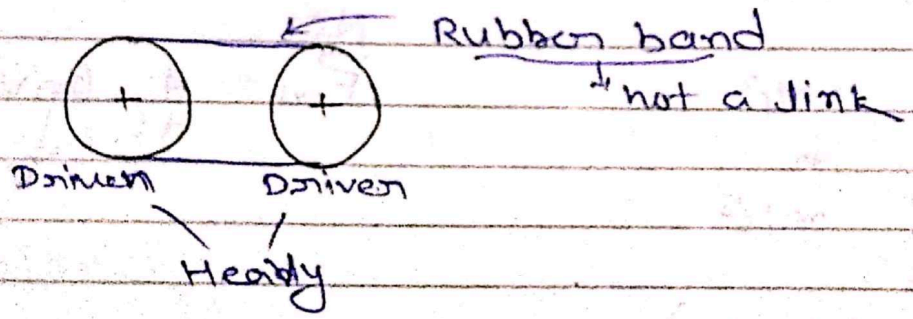
$\mu = \frac{N\text{-}s}{m^2}$

* Simple Mechanism -

* Kinematic link or Element -
"Every part of a m/c which is having some relative motion w.r.to some other part will be known as kinematical link or element".

It is necessary for the link to be a resistant body so that it is capable of transmitting power & motion from one element to the other element.

A link must be a resistant body.

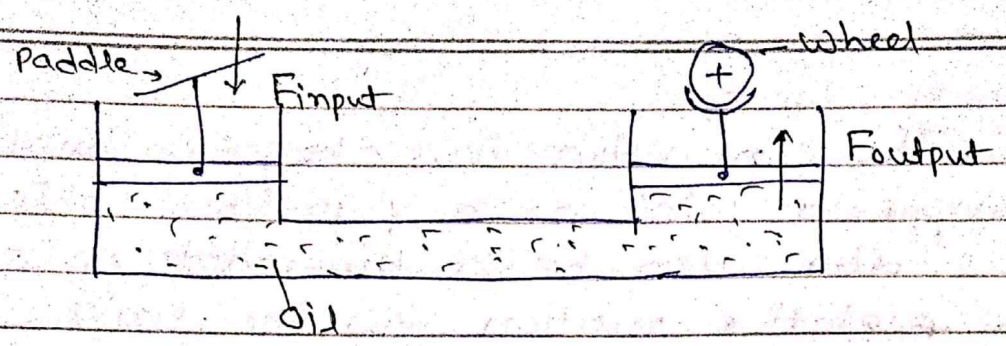


* Types of links -

1. Rigid link - Deformation are negligible.
   eg - Crank, piston, C.R, cylinder

2. Flexible links - Deformat^n is there but are in permissible limit.
   eg - Belt, rope, chain.

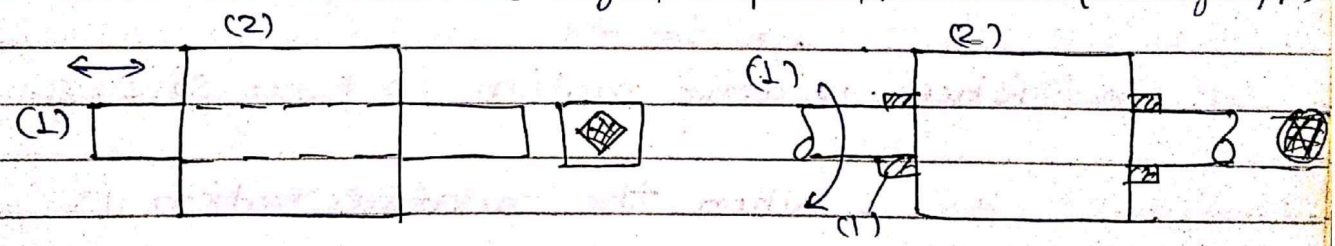
3. Fluid link - When the power is transmitted bcoz of fluid pressure
   eg. Hy. lift, Hy. crane, Hy. jack
   Hy. brake



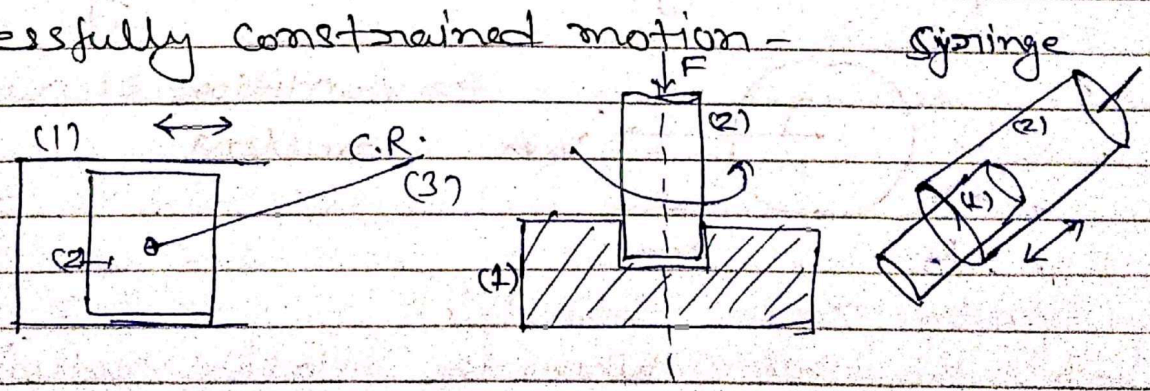
* Different types of Relative motion -

For a relative motion system is having two links
(Link 1, link 2)

1. Completely constrained motion - Constrained motion
   * Desired motion (single/unique I/P → unique/single O/P)

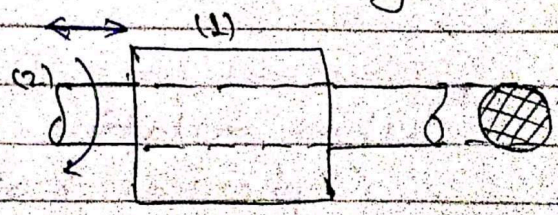


2. Successfully constrained motion -



Shaft in foot step bearing

3. Incompletely constrained motion - Unconstrained motion



* Kinematic pair -

" The connection b/w the two links is always a joint or a pair but this pair is ~~also~~ also be a kinematic pair if the relative motion b/w the links is a constrained motion."

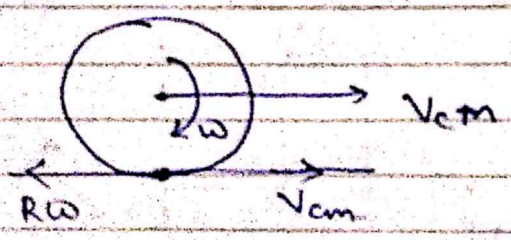
* Different types of kinematic pair -

A) Based on type of relative motion -

- Turning pair (Revolute pair, pin joint)
  When relative motion is pure turning

- Sliding pair (Prizmatic pair)
  When relative motion is pure sliding.

- Rolling pair - When the relative motion is pure rolling.
  pure rolling → without slipping



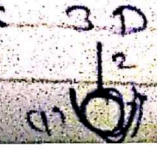
Condition of pure rolling

$R\omega = V_{cm}$

↑ Inde.

- Screw pair - When the relative motion ~~ov~~ is over the threads. (Nut bolt)

- Spherical pair (Ball in socket ~~pa~~ joint)
  When relative motion is 3D rotatⁿ.
  (Spherical motion)



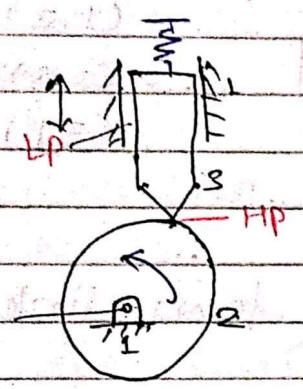
B) A/C to type of Contact -

Lower pair - Surface contact

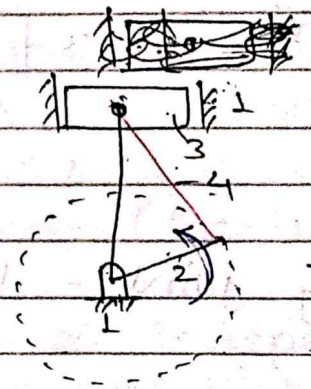
Higher pair - point / line contact

Wrapping pair - When one link is wrapped over other link - eg Belt pulleys

1 Hp ≡ 2 LP



Cam & Follower



To eliminate Hp

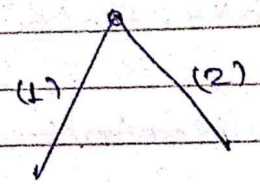
C) A/C to type of closure -

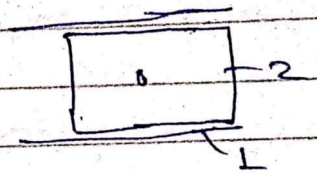
- Self closed pair (closed pair)
  Permanent contact

- Forced closed pair (open pair)
  Forceful contact
  eg - Hp in cam & follower, door closing mechanism, automatic clutch operating system

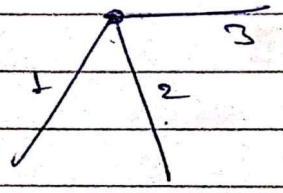
* Different types of joints.

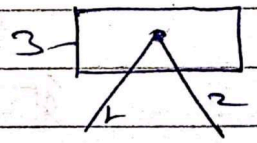
1. Binary joint - When two links are connected





2. Ternary joint - When three links are connected.

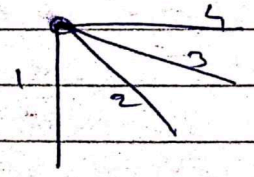




(1,2) - B
(2,3) - B
(3,1) - B
} Independent

1 T ≡ 2 B

3. Quaternary joint - When four links are connected.



1 Q ≡ 3 B

* Kinematic chain -

"If all the links are connected in such a way such that 1st link is connected to the last link in order to ~~eto~~ get the close chain & if all the relative motions in this closed chain are constrained, then such a chain is known as kinematic chain."

Kinematic chain - (constrained chain)

↓ To use this chain one link is fixed

Mechanism → which can give desired o/p w.r.to given I/P

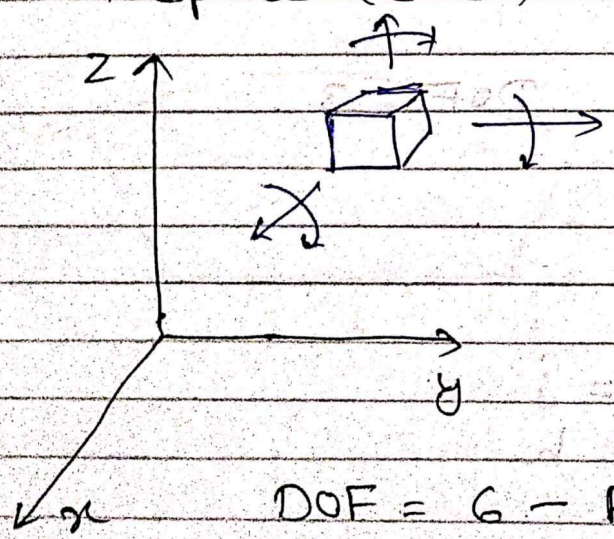
↓ utilish it

Machine - Desired o/p is obtained.

* Degree of freedom - (Mobility) -

" The min^m no. of independent variables required to define the position or motion of the system is known as DOF of the system."

In space (3-D.)

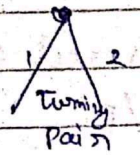
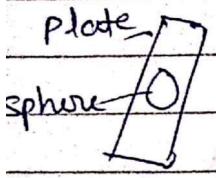


3T + 3R

DOF = 6 - Restraints

There motion are not possible.

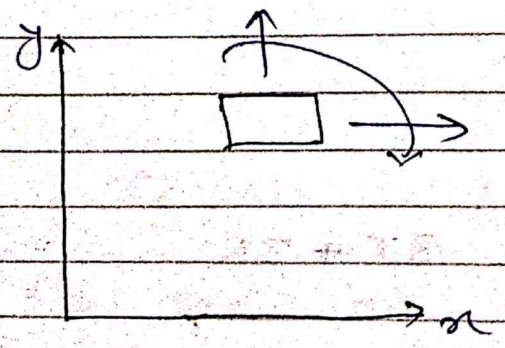
Restraints are always seen in the form of pair.

Pair	Restraints	DOF
	3T+2R = 5	6-5=1
	1-T = 1	6-1=5

LP → 1 DOF pair (SP pair DOF = 3)

HP → 2 DOF pair.

* Aim - To find out the DOF of 2D planar mechanism.



Max^m no. of motion = 3

DOF = 3

No. of link = L
No. of binary joint = j
No. of higher pair = h

One link fixed

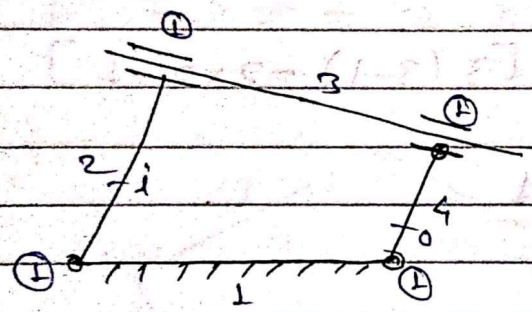
$$\boxed{F = 3(L-1) - 2j - h}$$ Kutzback's equ^n

No. of max^m motions in 2D planar mechanism.

$$F = [3(L-1) - 2j - h] - F_r$$ (Redundent)

No of those motion which are not part of mechanism

1.

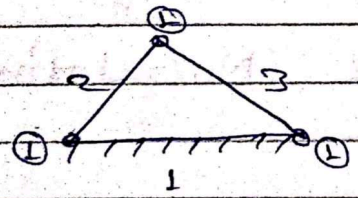


$L = 4$, $j = 4$, $h = 0$

$F_r = 1$

$F = (3(4-1) - 2\times4 - 0) - 1$

$F = 0$

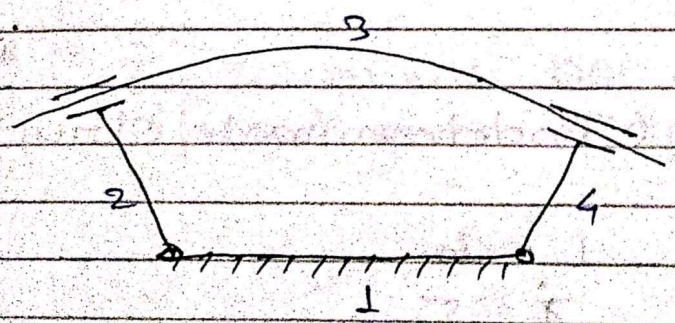


$L = 3$, $j = 3$, $h = 0$

$F = [3(3-1) - 2\times3 - 0]$

$F = 0$

2.

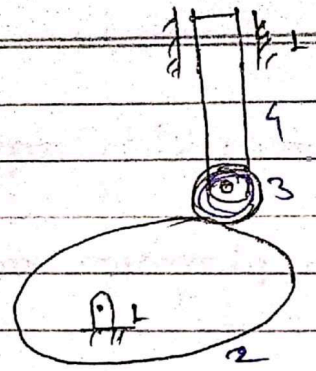


$L = 4$, $j = 4$, $h = 0$

$F_r = 0$

$F = 1$

3.



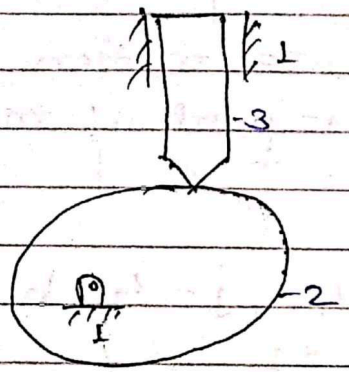
$L = 4$, $j = 3$

$h = 1$, $F_{\Rightarrow} = 1$

$F = [3(4-1) - 2\times3 - 1] - 1$

$F = 1$

4.



$L = 3$, $j = 2$

$h = 1$, $F_{\Rightarrow} = 0$

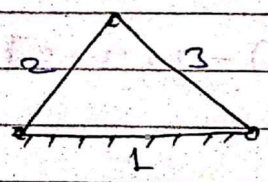
$F = [3(3-1) - 2\times2 - 1]$

$F = 1$

* If $F = 0$

Frame / Structure (No relative motion)

eg -

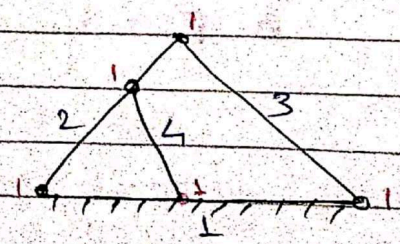


$F = 0$

* If $F < 0 \Rightarrow -1, -2 \ldots$

Super structure (Indeterminate structure)

eg -



$L = 4$

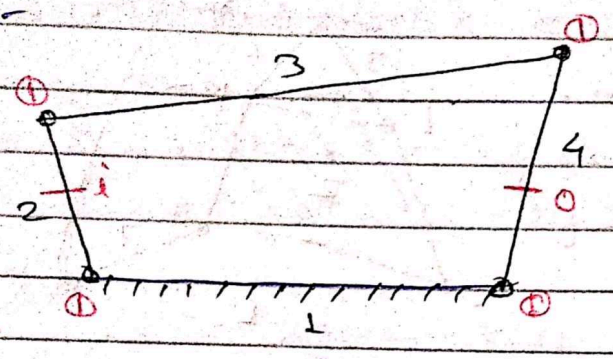
$j = 5$

$h = 0$

* $F = -1$

* If $F = 1$
↓ Kinematic chain.

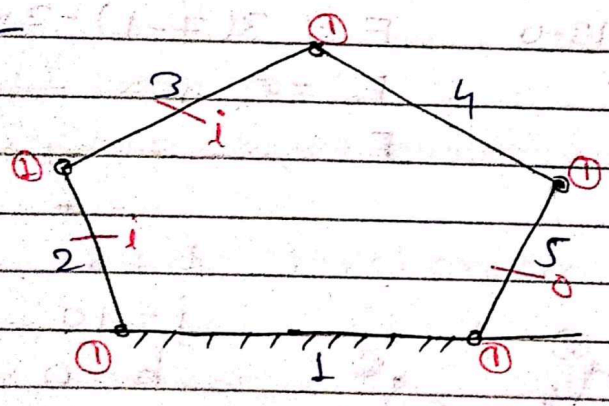
eg -



$L = 4$
$j = 4$
$h = 0$
$\boxed{F = 1}$

* If $F > 1 \Rightarrow 2, 3, 4 \ldots$
↓ Unconstrained chain

eg -

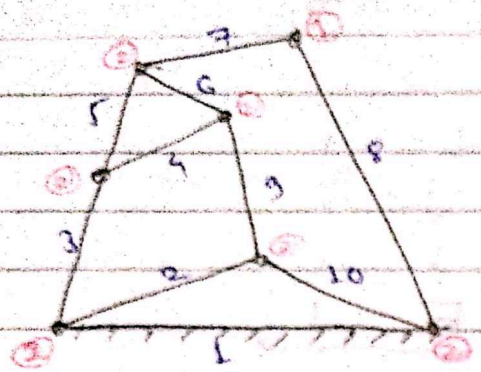


$L = 5$
$j = 5$
$h = 0$
$\boxed{F = 2}$

→ "DOF is the no. of inputs required to get constrained o/p in the chain."

* [To determine DOF → Fix one link always, if not given, Assume one link fix.]

*



$L = 10$

$j = 13$

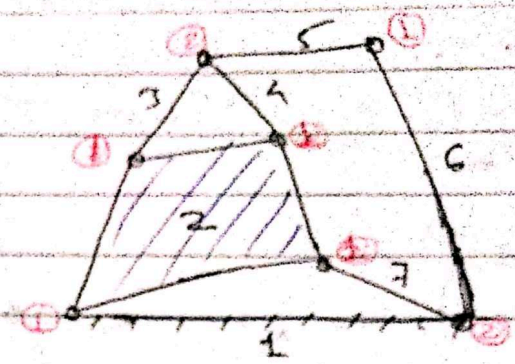
$h = 0$

$F = 3(10-1) - 2\times13 - 0$

$F = 1$

K.C.

*



$L = 7$

$j = 9$

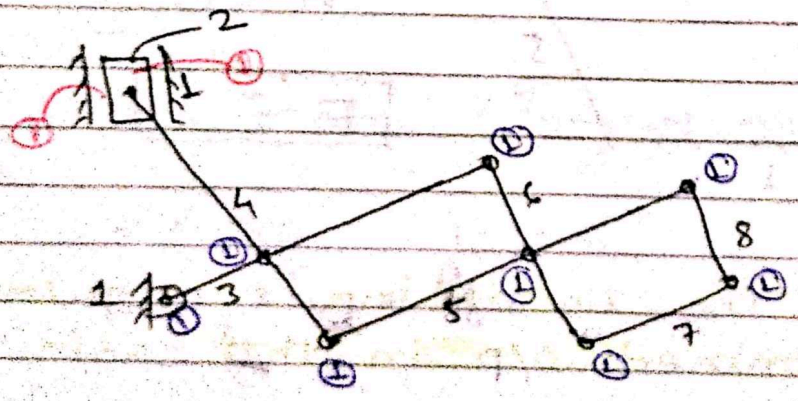
$h = 0$

$F = 3(7-1) - 2\times9 - 0$

$F = 0$

F

*



$L = 8$

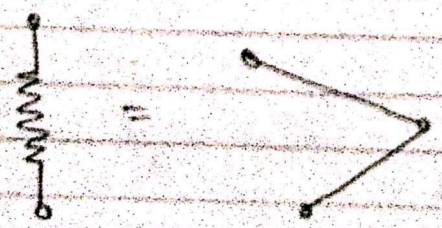
$j = 10$

$h = 0$

$F = 1$

* Spring as a link -

↑ Link of flexible length



* Grubler's equ^n -

Those mechanism in which

→ $F = 1$ & $h = 0$

Applied Kutzback's equ^n.

$$F = 3(L-1) - 2j - h$$
$$1 = 3L - 3 - 2j - 0$$

$$\boxed{3L - 2j - 4 = 0} \rightarrow \text{Grubler's equ}^n.$$

$(3L) \rightarrow$ even

$(L)_{always} \rightarrow$ even

$(L)_{min} = 4$

First mechanism with only Lower pair.

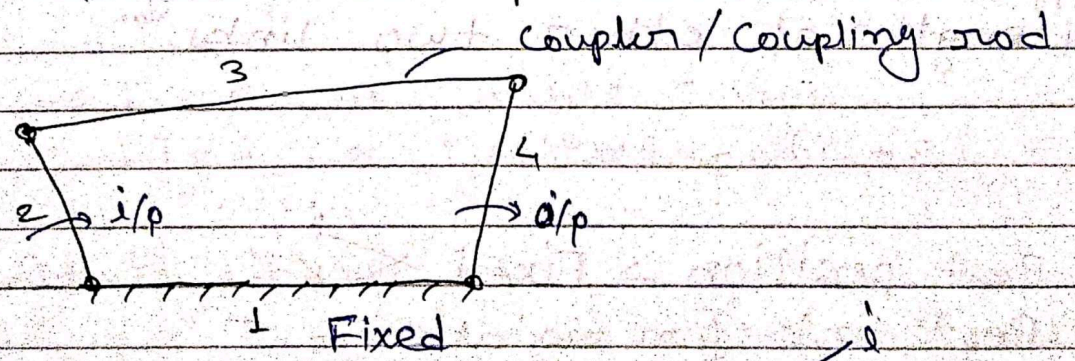
↓

Simple Mechanism

* Four bar Mechanism -

(Quaderic cycle mechanism)

* 4 links + 4 TP



Best position → Fixed < i, o

Input / output
↳ Complete rotation - Crank
↳ partial rotation - Rocker / lever

* Inversions - Mechanisms which are obtained by fixing one by one different links.

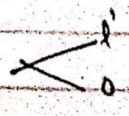
1. Double crank mechanism.
2. Crank ⟷ Rocker - " -
3. Double Rocker / lever - " -

Note - If no. of links = L
No. of inversion ≤ L

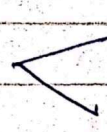
* Grashof's Law -

" For the continuous relative motion b/w the no. of links in ~~the~~ a mechanism the ~~sa~~ Σ of the length of shortest & longest links should not be greater than Σ of lengths of other two links."

$$(S+L) \leq (P+q)$$

Best position → Fixed 
Best link for rotation = S

Case 1. If $(S+L) < (P+q)$
Law is satisfied.

1. S - fixed  = Double crank

2. S - adjacent to fixed = crank - rocker

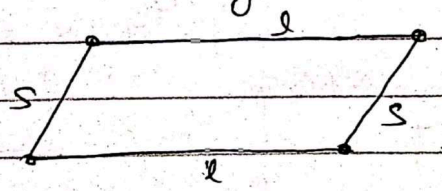
3. S - coupler → Double Rocker

2. If $(S+L) = (P+q)$
Law is satisfied
[Not having pair of equal links]
eg. 2, 5, 3, 4

* Same as previous. (L-B)

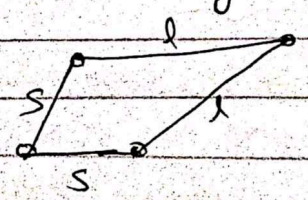
3. If $(S+L) = (P+q)$, Law is satisfied
Having ~~equal~~ pair of equal links
eg. 2, 2, 5, 5

(i) Parallelogram linkage:



S → Fixed → Double crank
L → Fixed → Double crank

(ii) Deltoid linkage -

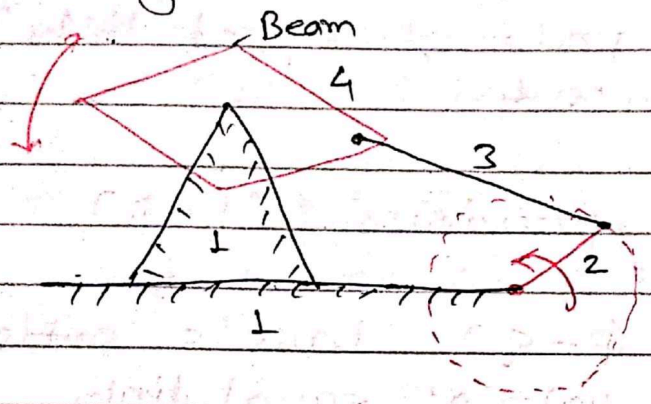


S - Fixed - Double crank
L - fixed - crank rocker

Case III if $(S+L) > (P+q)$
Law is not satisfied
↓
Double Rocker – always.

* Some practical apps of 4 bar Mechanism –

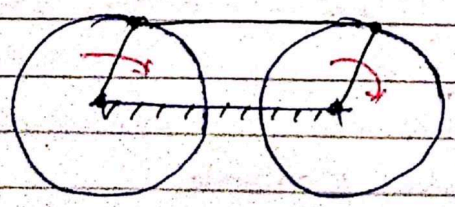
1. Beam engine Mechanism –



Rotat^n → oscillat^n
Crank → Rocker

eg – Stiching m/c

2. Coupling rod of locomotive –
(parallelogram linkage)



Trailing wheel

Driving wheel

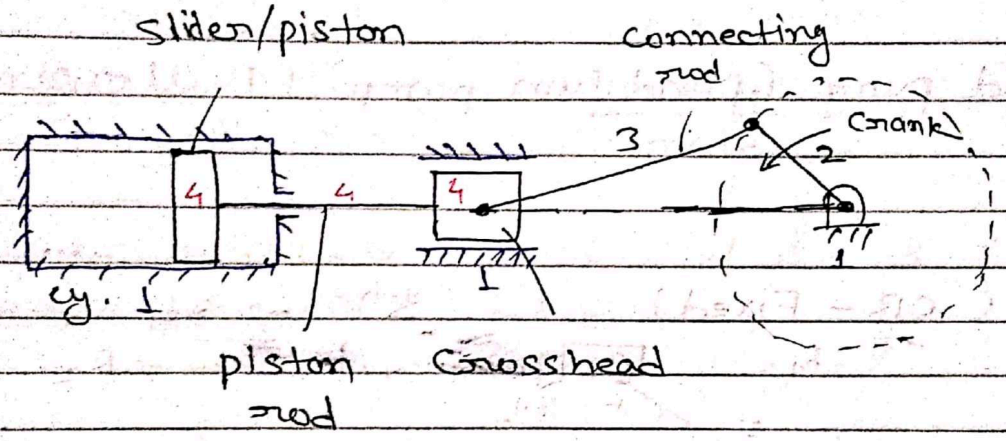
* Single slider crank Mechanism -

4 link + 3TP + 1SP

Basic single slider crank mechanism -

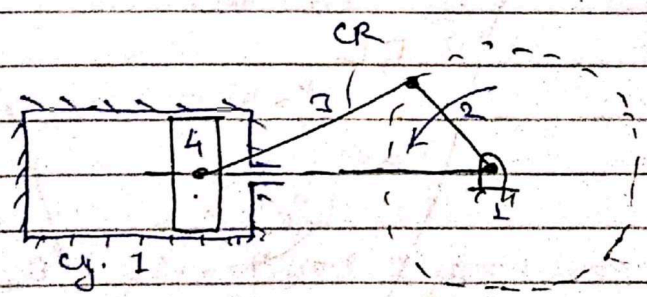
I- Inversion - (cylinder - fixed)

(EC)



I-Inversion
cy. fixed

(IC)



Rot. → Recipr.
(crank) (piston)

i → o — Reciprocating compressor

o ← i — Rec. engine.

II - Inversion - Crank fixed.

Whitworth QRMM

Rotary IC engine Mechanism
(GNOME engine)
(aeroplane). past.

III - Inversion - CR - Fixed

crank & slotted lever QRMM (Best)

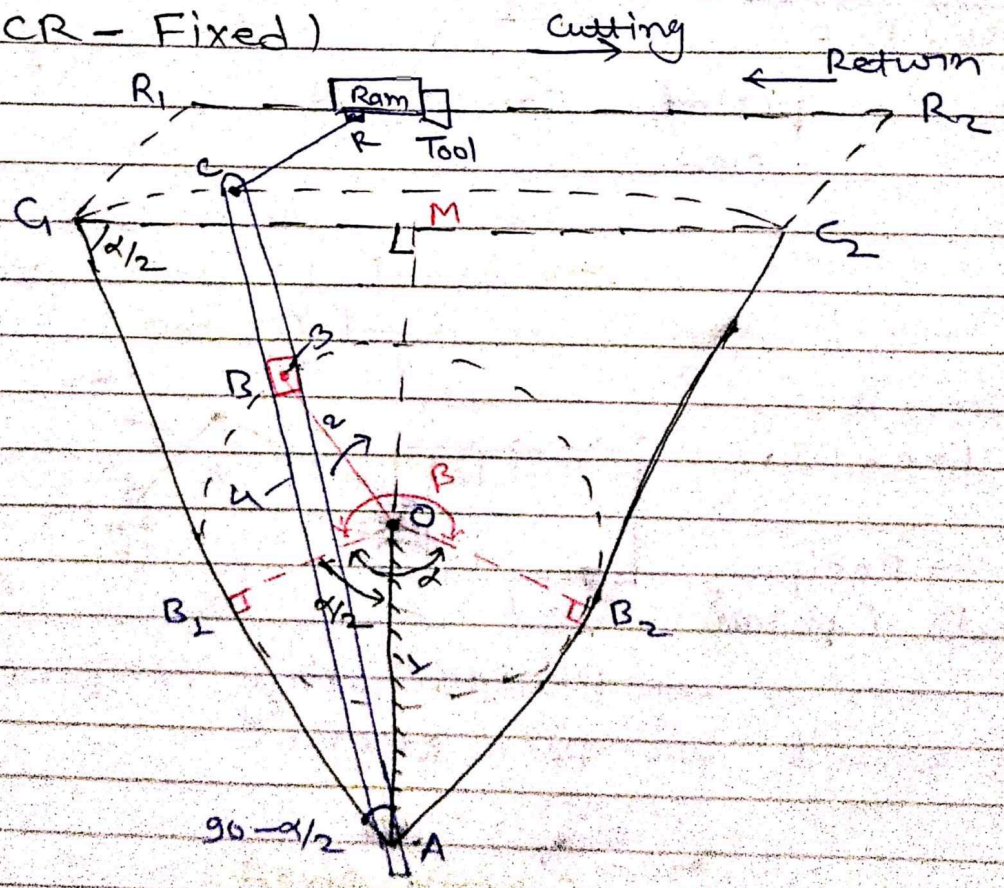
oscillatⁿ cyl. engine Mechanism

IV - Inversion - Slider fixed

Hand pump (pendulum pump) (Bull engine)

* Crank & slotted lever QRMM -

(CR - Fixed)



β = Cutting stroke angle

α = Return stroke angle

$\alpha + \beta = 360°$

$\alpha < \beta$

$(\text{time})_{\text{Return}} < (\text{time})_{\text{cutting}}$ (QRMM)

→ Quick Return Ratio (QRR) $= \dfrac{(\text{time})_{\text{cutting}}}{(\text{time})_{\text{Return}}} = \beta/\alpha$ (always >1)

Stroke length $= R_1R_2$

$= C_1C_2$

$= 2(C_1M)$

$= 2(AC_1)\cos\alpha/2$

$= 2(AC_1)\dfrac{(OB_1)}{OA}$

$= \dfrac{2(AC)(OB)}{OA}$

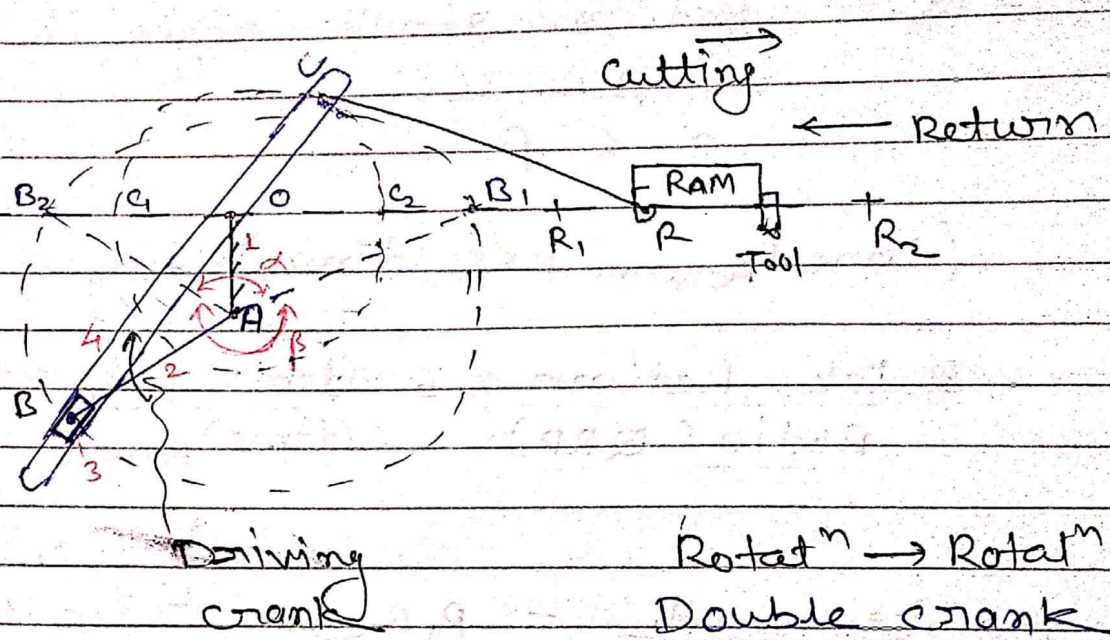
→ Stroke $= \dfrac{2(\text{length of slotted bar}) \times (\text{length of crank})}{\text{Length of CR}}$

Rot^n ⟶ oscillat^n

↓ ↓

Crank Rocker.

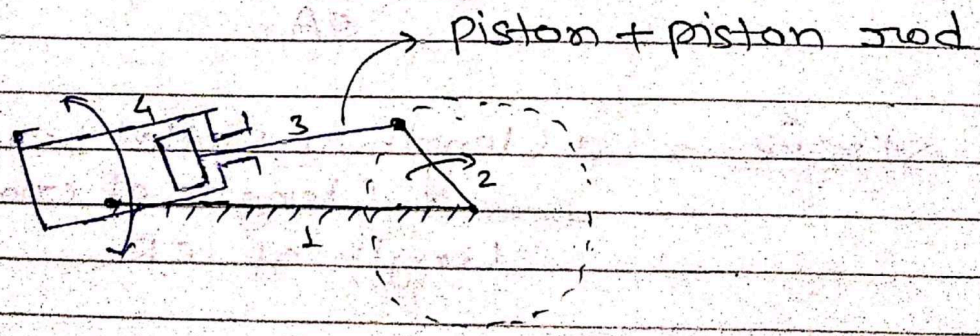
* Whitworth - QRMM -
(Crank fixed)



Rotat^n → Rotat^n
Double crank

stroke = R_1R_2
= C_1C_2
= 2 (OC)

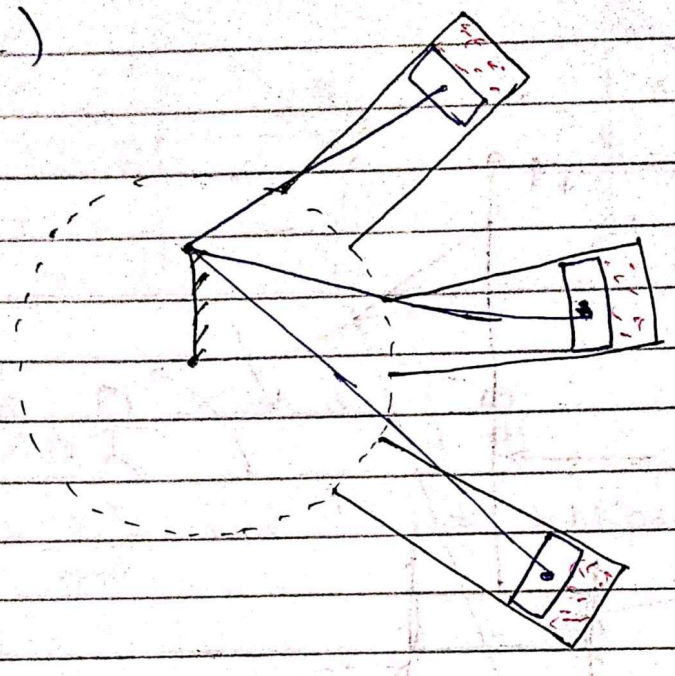
* Oscillating cy. Engine Mechanism -
(CR fixed)



Rotat^n ⟷ oscillat^n
Crank ⟷ Rocker

Rotary IC engine Mechanism - (GNOME Engine)

(Crank fixed)



When combⁿ takes place inside the cy. I/p

↓

Input force comes on piston

↓

This I/p force is transmitted to CR

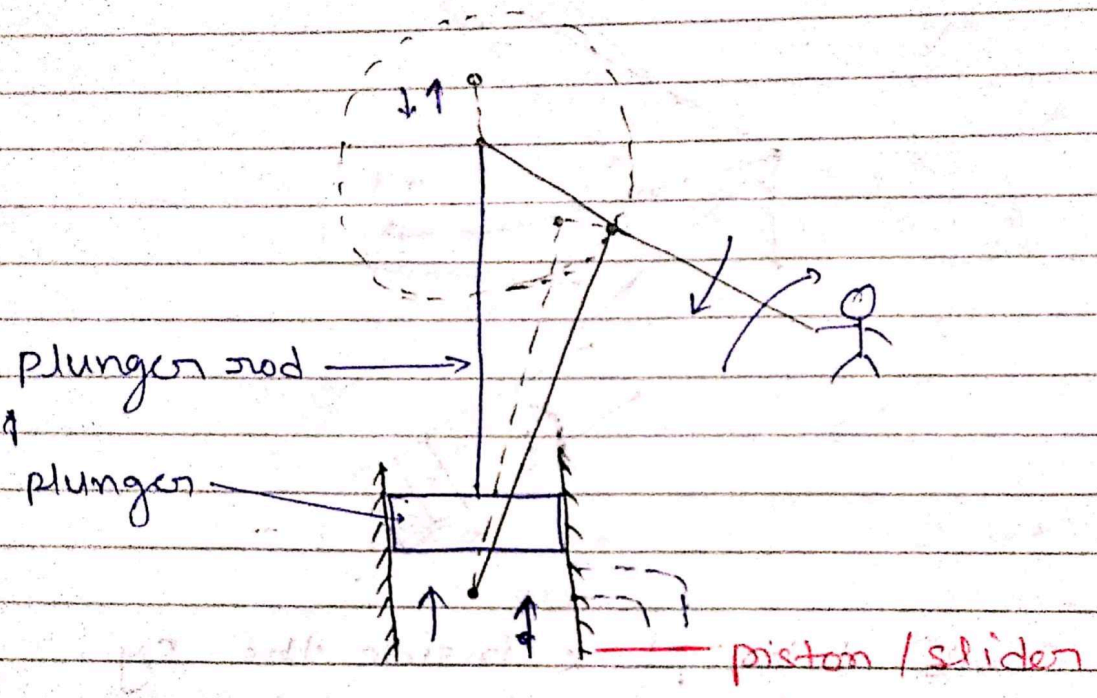
↓

CR & piston rod both rotate oscillate

↓

Cy block rotate — O/p

* Hand pump (Pendulum pump) (Bell Engine
(slider fixed)



Machine	Mechanism
1. It converts available energy into useful work	It converts or modify motion.
2. The dimension & shape are very important parameter for constructing machine.	The dimension & shape are not so important for constructing mechanism.
3. While designing ~~a~~ machine, strength requirements are taken into consideration.	While designing mechanism the required motions are taken into consideration.
4. eg. steam engine, pump. IC engine.	Single slider, double slider mechanism, lock.
5. Every machine is either a mechanism or a combination of more than one mechanism.	Every mechanism is not necessarily a machine.
6. It is used to transmit force.	It is not used to transmit the force.

Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} \quad \text{or} \quad \frac{360° - \beta}{\beta}$$

Note. In order to find the length of effective stroke $R_1 R_2$, mark $P_1 R_1 = P_2 R_2 = PR$. The length of effective stroke is also equal to 2 PD.

Example 5.1. *A crank and slotted lever mechanism used in a shaper has a centre distance of 300 mm between the centre of oscillation of the slotted lever and the centre of rotation of the crank. The radius of the crank is 120 mm. Find the ratio of the time of cutting to the time of return stroke.*

Solution. Given : $AC = 300$ mm ; $CB_1 = 120$ mm

The extreme positions of the crank are shown in Fig. 5.28. We know that

$$\sin \angle CAB_1 = \sin(90° - \alpha/2)$$

$$= \frac{CB_1}{AC} = \frac{120}{300} = 0.4$$

$$\angle CAB_1 = 90° - \alpha/2$$

$$= \sin^{-1} 0.4 = 23.6°$$

$$\alpha/2 = 90° - 23.6° = 66.4°$$

$$\alpha = 2 \times 66.4 = 132.8°$$

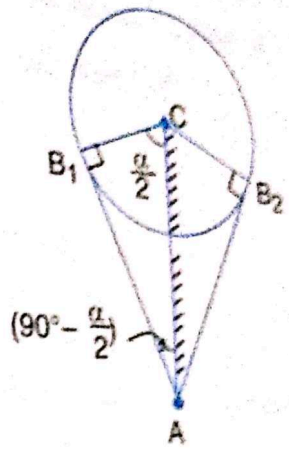


Fig. 5.28

We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - \alpha}{\alpha} = \frac{360° - 132.8°}{132.8°} = 1.72 \text{ Ans.}$$

...ation mechanism, the distan...

M-J-2015 / June 2014.
A-M-2017. - given - Crank & slotted lever QRMM

Length of C.R. = 6.5 cm = OA
Length of Crank = 2 cm = OB = OB_1

Time ratio = ? , Length of stroke = ?

For extreem position. In $\Delta\ OB_1A$, $\angle OB_1A = 90°$
considering Base - OB_1

$\therefore$

$$\cancel{\tan \alpha/2} \quad \cos \alpha/2 = \frac{OB_1}{OA} = \frac{2}{6.5}$$

$$\alpha/2 = 72° \Rightarrow \alpha = 144°$$

Now, $\beta = 360° - \alpha = 216°$

$\therefore$ QR ratio $= \frac{\beta}{\alpha} = \frac{216}{144} = 1.5$ Ans

not given so assume data.

Length of stroke = $\frac{2 \times (\text{Length of slotted bar}) \times (\text{Length of crank})}{\text{Length of CR}}$.

put values & get Answer.

3. Friction

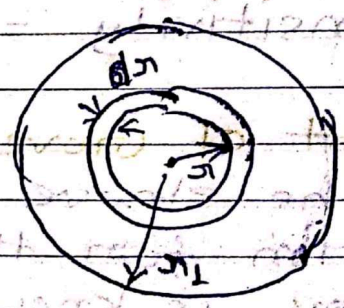
- Pivot & Collar friction - B/w shaft & bearing
- Power lost in friction Assuming -

1. Uniform pressure theory - For new clutch

Let P = Pressure intensity (N/m^2) Pa
$W = mg$ = weight in (N)
r_1 = external radius
r_2 = internal radius
μ = Coefficient of friction

T = Torque / Resisting torque

Consider an ~~sec~~ element of thickness dr at a distance of r from centre.



(bearing)

∴ Area of element
$= 2\pi r\, dr$

weight on bearing $= P \times A$
$W = p \cdot 2\pi r\, dr$

& Frictional torque on element $= \mu W r$
$= \mu \cdot p \cdot 2\pi r\, dr \cdot r$

$= \mu p \cdot 2\pi r^2\, dr$

$\therefore$ Total frictional torque on bearing, $T = \int_{r_2}^{r_1} \mu p \cdot 2\pi r^2 \, dr$

If p = constant, then

$$W = \int_{r_2}^{r_1} p \cdot 2\pi r \, dr$$

$$W = 2\pi p \frac{(r_1^2 - r_2^2)}{2} = \pi p (r_1^2 - r_2^2) \quad \text{—(1)}$$

&

$$\text{Torque } T = \int_{r_2}^{r_1} \mu p \, 2\pi r^2 \, dr$$

$$= \mu p \, 2\pi \frac{(r_1^3 - r_2^3)}{3}$$

$$T = \frac{2}{3} \mu p \pi (r_1^3 - r_2^3) \quad \text{—(2)}$$

From eq^n (1) $\pi p = \dfrac{W}{(r_1^2 - r_2^2)}$

put in eq^n (2), we get

$$T = \frac{2}{3} \mu W \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)}$$

$$\boxed{T = \mu W R_m}$$

where $R_m = \dfrac{2}{3} \dfrac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)}$

Ans

2. Uniform wear theory – for old clutch wear is product of press. & radius.

$\therefore$ in this pr = constant = c

$$\therefore W = \int_{r_2}^{r_1} 2\pi c \, dr = 2\pi c (r_1 - r_2)$$

$$\& \quad T = \int_{r_2}^{r_1} 2\pi c \mu r \, dr$$

$$= 2\pi c . \mu \frac{(r_1^2 - r_2^2)}{2}$$

$$T = \mu \pi c (r_1^2 - r_2^2)$$

$$T = \frac{\mu W}{2} \frac{(r_1^2 - r_2^2)}{(r_1 - r_2)}$$

$$= \frac{\mu W}{2} \frac{\cancel{(r_1 - r_2)} (r_1 + r_2)}{\cancel{(r_1 - r_2)}}$$

$$T = \frac{\mu W}{2} \frac{(r_1 + r_2)}{2}$$

$$\boxed{T = \mu W R_m}$$

where $R_m = \frac{(r_1 + r_2)}{2}$

$c = p_{max} r_{min} = p_{min} r_{max}$

$n = \frac{\text{Total load}}{\text{load per collar}}$.

For Numerical - steps

1. Write given data

2. For $(P = C)$; $W = \pi p (r_1^2 - r_2^2)$

& $T = \mu W R_m$; $R_m = \frac{2}{3}\frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)}$

3. For $(Pr = C)$; $W = 2\pi C (r_1 - r_2)$

$T = \mu W R_m$; $R_m = \left(\frac{r_1 + r_2}{2}\right)$

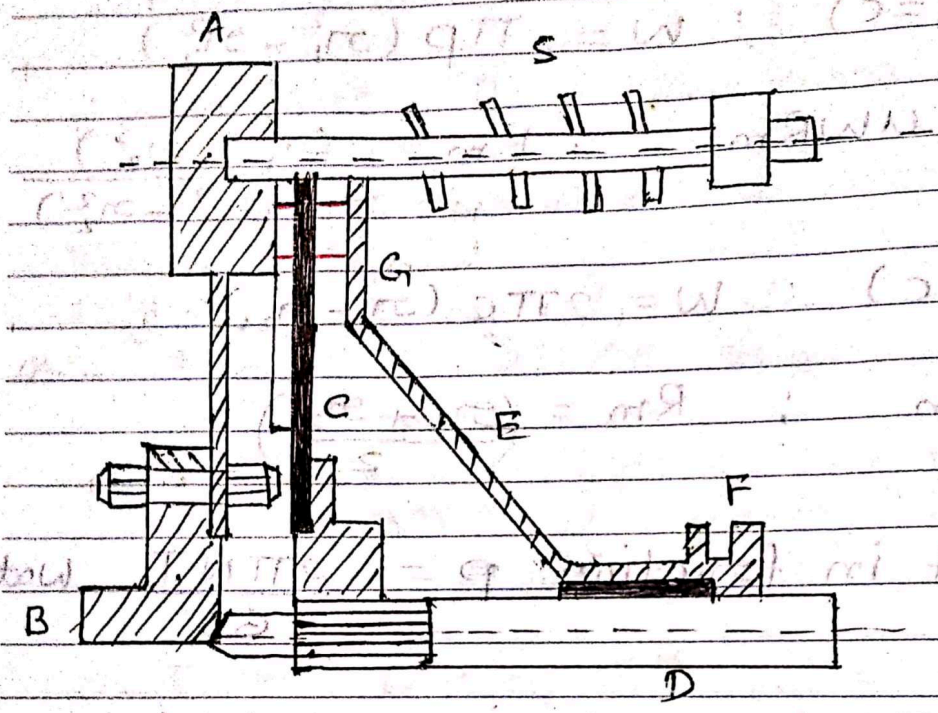
4. Power lost in friction $P = \frac{2\pi N T}{60}$ Watt

- Clutches - It is a device used to connect or disconnect the driving shaft & the driven shaft when desired by operator or driver.

These are widely used in automobiles. These are called friction clutches as friction is used.

- clutch
  - Plate or disc clutch
    - single plate
    - multiplate
  - conical clutch

- single plate clutch -



The single plate clutch is shown in fig. below. The flywheel A is bolted to a ~~forme~~ flange on the ~~sha~~ driving shaft B. The plate C is fixed to a boss free to slide axially along the driven shaft D, but by means of splines, it is compelled to revolve with the shaft D. Two rings G of special friction material are riveted to A & E.

The pressure plate E is connected to bush, so as to revolve freely on

the driven shaft. A no. of spiral springs are arranged around the the clutch, so as to provide axial thrust b/w the friction surfaces.

- Clutch is a device which transmits power b/w shaft which are running at different speed. Clutch is used b/w engine crank shaft & gear shaft. With the help of clutch the transmission of power from driving shaft to driven shaft can be done, whereas the driving shaft is always at running condition.

- When the withdrawal sleeve F is displaced to right, the spring is compressed. There is no axial pressure b/w friction surfaces & flywheel. A revolves freely while plate C & shaft D remains at rest. This process is called declutching.

When force is removed from sleeve the spring forces the plate E against 20 SUNDAY the friction b/w ~~conta~~ plate & ring transmits a torque to shaft D. & the shaft D revolves.

used in - ~~cars~~, trucks & buses. (Required Large area)

233-132. Multiplate clutch -

Multiplate clutch is similar to single plate clutch except the no. of surfaces is increased. Due to ~~no~~ increased contact area, they have more power transmission capacity.

Displace-ment of withdrawal sleeve F to the right removes the axial load from the disc & allow the outer casing to revolve freely while the shaft D remains at rest.

The friction torque b/w each pair of contact surfaces is given by -

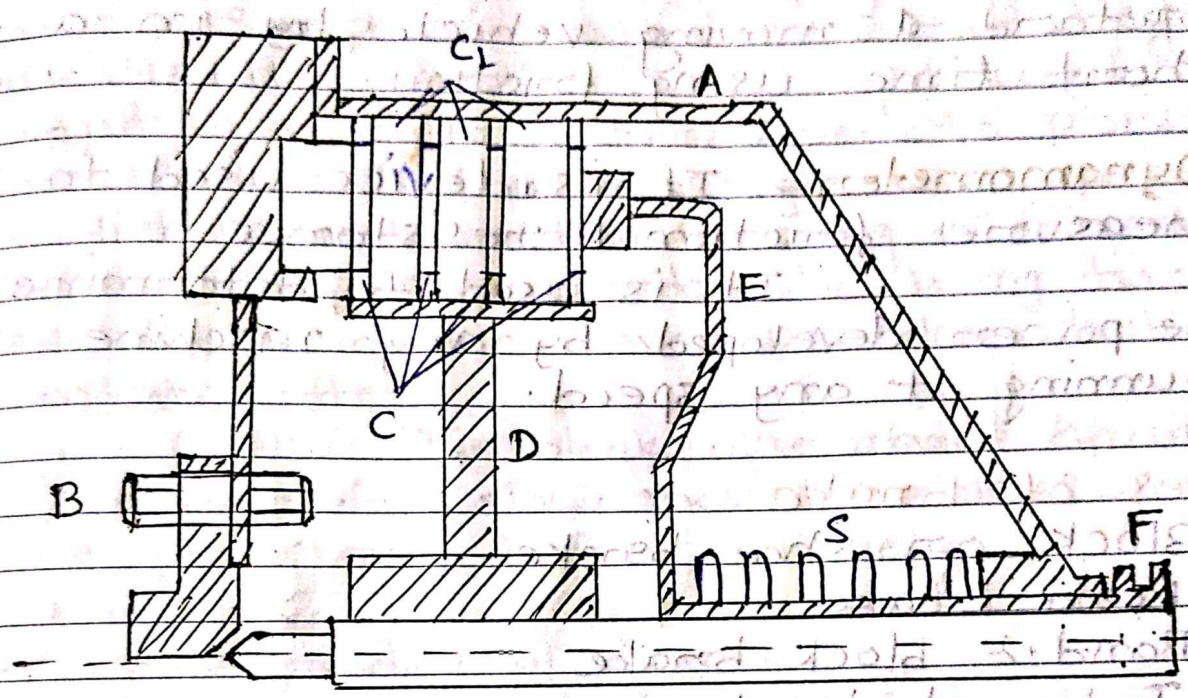
$$T = \mu W \frac{(r_1 + r_2)}{2}$$

& Total torque is given by

$$T = n \mu W \frac{(r_1 + r_2)}{2}$$

where $W =$; $\mu =$; , $n =$, $r_1 =$; $r_2 =$?

When the force is removed from sleeve the engagement of plates & spring occurs & torque is transmitted to shaft so as to revolve the shaft.



- For Multiplate clutch -

$$T = n\mu W R_m \quad \text{where } n = n_1 + n_2 - 1$$

n_1 = no. of plate in driving shaft
n_2 = — " — driven shaft

- If both sides of plates are active : then
  ↳ For single plate clutch

$$T = 2\mu W R_m$$

- n = no. of pairs in contact.

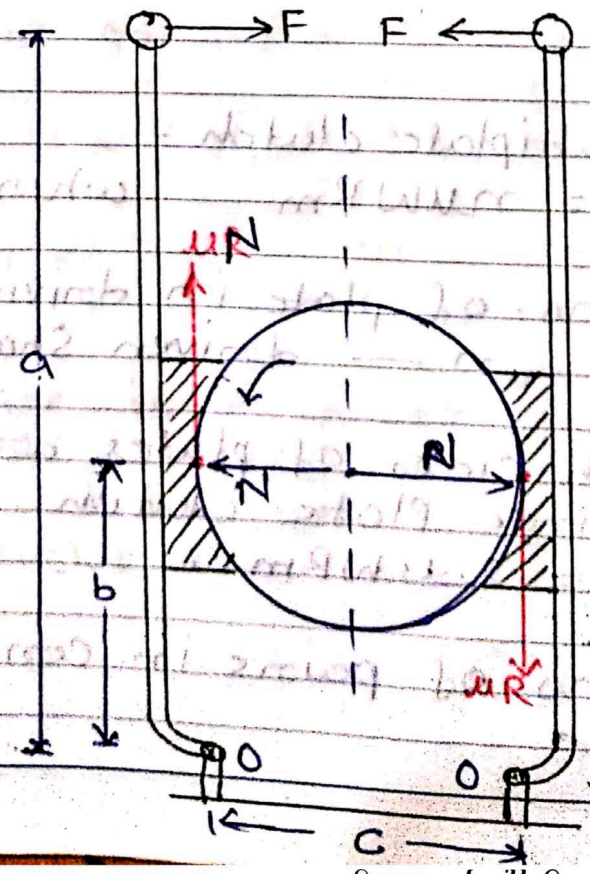
- **Brake** - It is a device used to stop or retard a moving vehicle ~~by~~ in a short time using friction.

- **Dynamometer** - It is a device used to measure frictional resistance.
  It is used to determine the power developed by any machine running at any speed.

Types of brake

1. Block or shoe brake
2. Band brake
3. Band & block brake
4. Internal brake.

1. Block brake -



It consists of a block having friction layer on it, is pressed against a rotating wheel called drum. A lever is used to increase the force required to press the block. But a single lever produces a side thrust on the bearing of drum shaft. This is prevented by using two levers. The double lever provides double braking effect.

The block are made of wood or rubber for slow moving vehicles & cast iron for heavy & fast vehicles.

Let r = radius of drum
μ = Coeff. of friction
F_R = Radial force applied on drum
N = Normal reaction on block
F = Force applied on lever
F_f = friction force = μN

T_B = Braking torque.

T_B = Frictional force × Radius
$T_B = \mu N \cdot r$

For calculation purpose, we take moment about O.

237-128

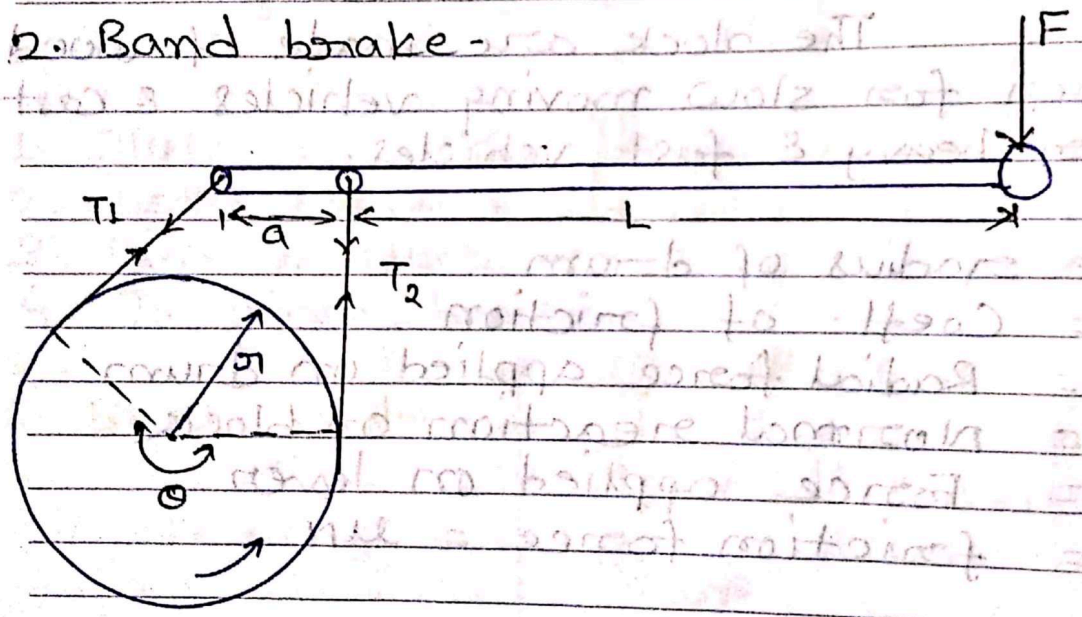
* $\frac{T_n}{T_0} = \left[\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right]^n$; n = no. of blocks
T_0 = Tension in slack ~~sight~~ side

$T_1, T_2 \ldots T_n$ = tension is tight side for block 1, 2, ... n.

μ = coeff. of friction

N = normal reaction. $\Rightarrow T_B = (T_n - T_0)\cdot r_m$

2. Band brake -



When the two ends of band are pined or tied by pivot, then the brake is called simple band brake.

Simple band brake can be self locked. The brake is said to be more effective when maximum braking force is applied with the least effort

F. The advantage of self locking is taken in hoists & conveyers where motion is permissible in only one direction.

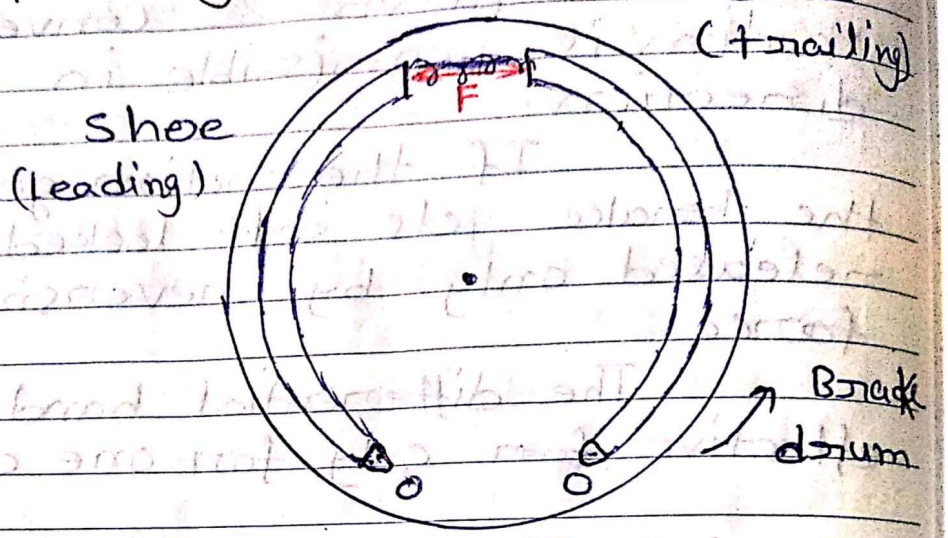
If the motion gets reversed, the brake gets self locked which can be released only by reversing the applied force.

The differential band brake is more effective ~~for~~ only for one dir^n of motion.

3. Band & block brake - This brake consists of numbers of blocks made of wood secured inside the flexible steel. The blocks are pressed against the drum when brake is applied.

*

4. Internal expanding shoe brake -



In all the two wheelers, automobiles & other vehicles now a days only internal expanding shoe brakes are used.

It consists of two semicircular shoes which are lined with a friction material.

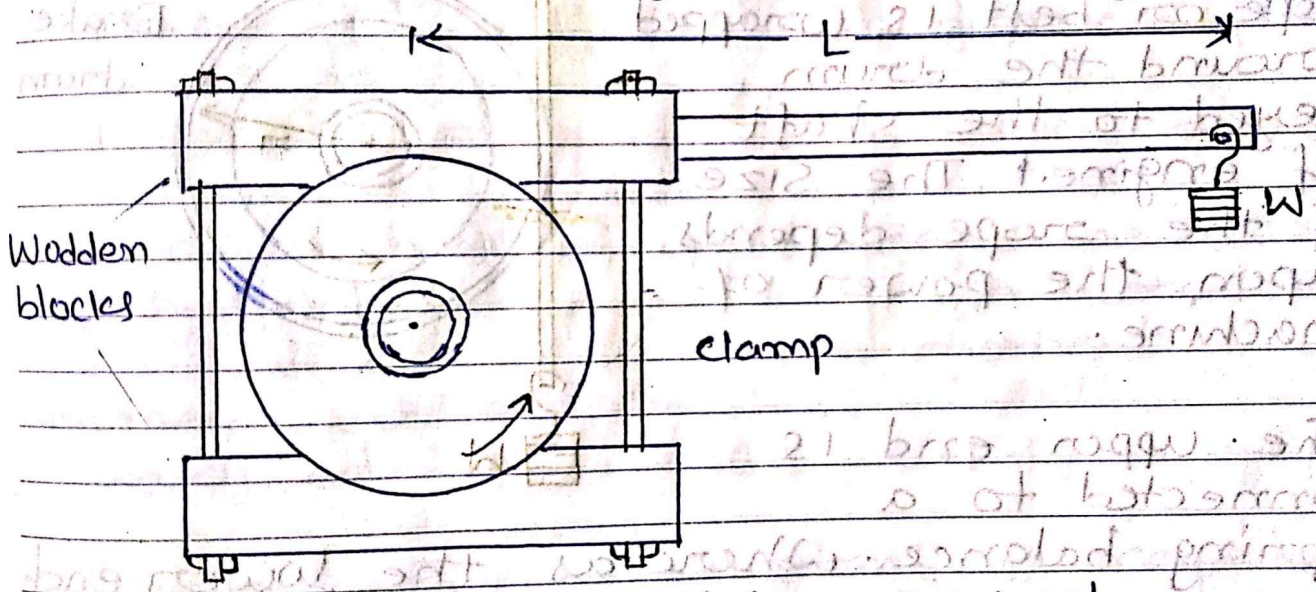
The shoes are pressed against the inner wall of drum when the brakes are applied. There will be a little gap b/w shoe & drum wall when brake is not applied.

Depending upon the direction of rotation one of shoe will act as leading & the other acts as trailing shoe.

The leading shoe is self-energising but trailing shoe is not.
Under any circumstances they should not be self locking

• Dynamometer :-

1. Prony brake dynamometer -



It is the simplest form of dynamometer. it consist of two wooden blocks clamped together on revolving pulley carrying a lever. The grip b/w blocks & drum can be varied by tightening or loosing the bolts. This can be used upto a speed of 1000 rpm & 100 kW of power.

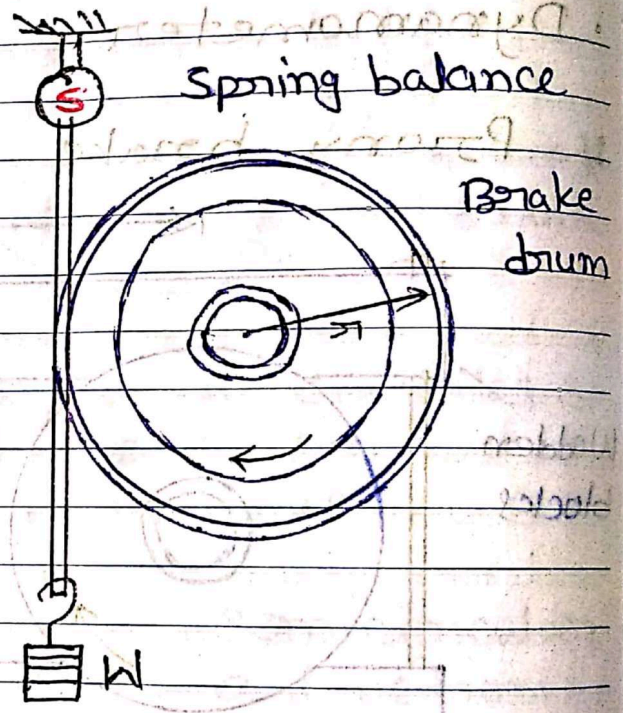
T_F = Frictional torque = Load (force) x distance

$T_f = W \times L$

& power $P = T_f \times \frac{2\pi N T_f}{60}$

• power is independent of drum size & el.

• 2. Rope brake dynamometer -



This is most commonly used dynamometer. Rope or belt is wrapped around the drum keyed to the shaft of engine. The size of the rope depends upon the power of machine.

The upper end is connected to a spring balance. where as the lower end ~~carry~~ carries a load.

power of machine

$P = \text{Torque} \times \text{ang. velocity } (\omega)$

$P = (W - S) \cdot r \times \omega$ Watt.

$\omega = \frac{2\pi N}{60}$

1 HP = 735 Watt
1 KW = 1.36 HP

Sliding Contact Bearings

Classification of Bearings

1. **<u>According to the direction of load to be supported.</u>**

The bearings under this group are classified as:

(*a*) Radial bearings

(*b*) Thrust bearings.

In ***radial bearings***, the load acts perpendicular to the direction of motion of the moving element as shown in the Figure 1 (a & b).

In ***thrust bearings,*** the load acts along the axis of rotation as shown in (*c*).

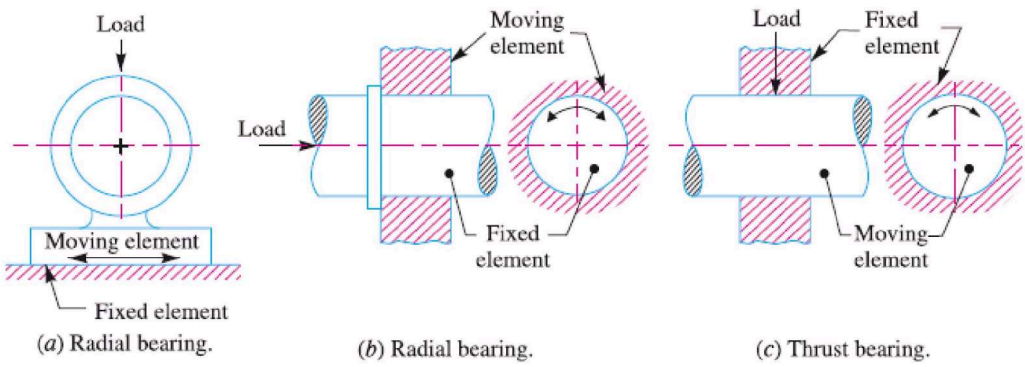


Figure 1

2. <u>According to the nature of contact.</u>

The bearings under this group are classified as:

(*a*) Sliding contact bearings,

(*b*) Rolling contact bearings.

In ***sliding contact bearings,*** as shown in the following Figure 2. (*a*), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as ***plain bearings.***

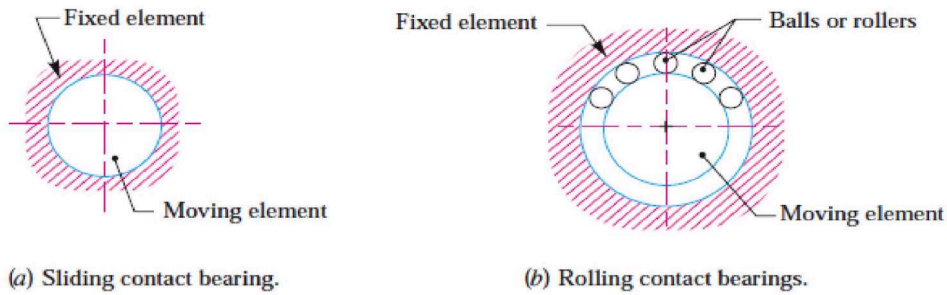
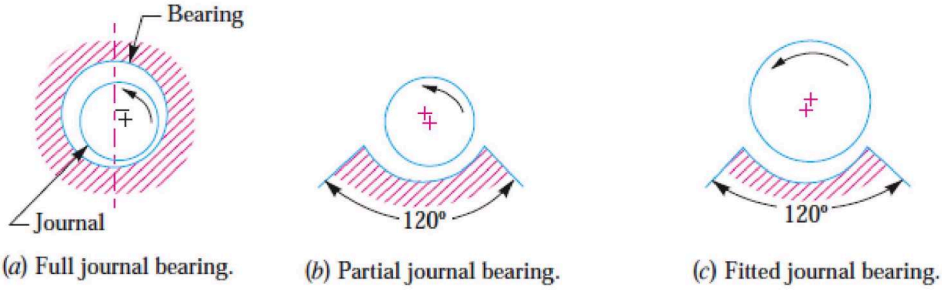


Figure 2

In ***rolling contact bearings,*** as shown in figure 2 (*b*), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.

Types of Sliding Contact Bearings

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in figure 3 (*a*), may be called ***slipper*** or ***guide bearings.*** Such type of bearings are usually found in cross-head of steam engines.



(*a*) Full journal bearing. (*b*) Partial journal bearing. (*c*) Fitted journal bearing.

Figure 3

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as ***journal*** or ***sleeve bearings.*** When the angle of contact of the bearing with the journal is 360° as shown in the above figure (*a*), then the bearing is called a ***full journal bearing.*** This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.

When the angle of contact of the bearing with the journal is 120°, as shown in figure 3 (*b*), then the bearing is said to be ***partial journal bearing.*** This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as ***clearance bearings*** because the diameter of the journal is less than that of bearing.

When a partial journal bearing has no clearance *i.e.* the diameters of the journal and bearing are equal, then the bearing is called a ***fitted bearing,*** as shown in figure 3 (*c*).

The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows:

1. <u>Thick film bearings</u>. The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as ***hydrodynamic lubricated bearings.***

2. <u>Thin film bearings</u>. The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other atleast part of the time. Such type of bearings are also called ***boundary lubricated bearings.***

3. <u>Zero film bearings.</u> The zero film bearings are those which operate without any lubricant present.

4. <u>*Hydrostatic or externally pressurized lubricated bearings*</u>. The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

<u>Hydrodynamic Lubricated Bearings</u>

We have already discussed that in hydrodynamic lubricated bearings, there is a thick film of lubricant between the journal and the bearing. A little consideration will show that when the

CHAPTER 27

Rolling Contact Bearings

27.1 Introduction

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction. Due to this low friction offered by rolling contact bearings, these are called ***antifriction bearings.***

27.2 Advantages and Disadvantages of Rolling Contact Bearings Over Sliding Contact Bearings

The following are some advantages and disadvantages of rolling contact bearings over sliding contact bearings.

Advantages

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

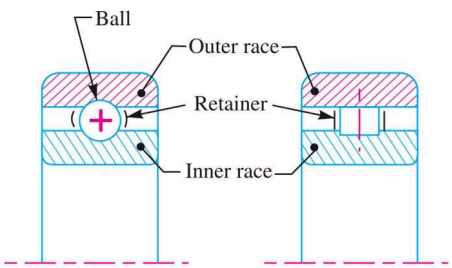
Disadvantages

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.

27.3 Types of Rolling Contact Bearings

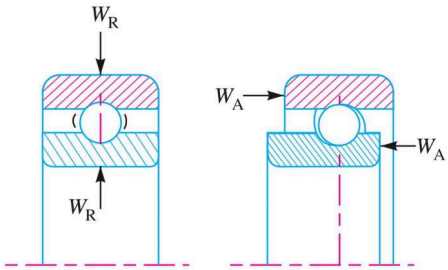
Following are the two types of rolling contact bearings:

1. Ball bearings; and **2.** Roller bearings.



(*a*) Ball bearing. (*b*) Roller bearing.

Fig. 27.1. Ball and roller bearings.



(*a*) Radial ball bearing. (*b*) Thrust ball bearing.

Fig. 27.2. Radial and thrust ball bearings.

The ***ball and roller bearings*** consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig. 27.1. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads.

The rolling contact bearings, depending upon the load to be carried, are classified as :

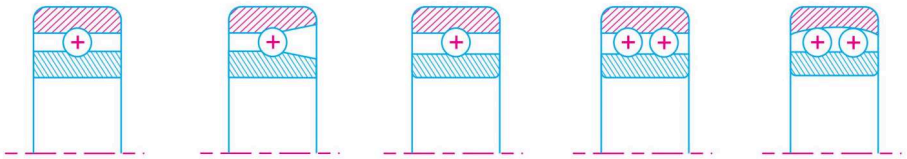
(*a*) Radial bearings, and **(*b*)** Thrust bearings.

The radial and thrust ball bearings are shown in Fig. 27.2 (*a*) and (*b*) respectively. When a ball bearing supports only a radial load (W_R), the plane of rotation of the ball is normal to the centre line of the bearing, as shown in Fig. 27.2 (*a*). The action of thrust load (W_A) is to shift the plane of rotation of the balls, as shown in Fig. 27.2 (*b*). The radial and thrust loads both may be carried simultaneously.

27.4 Types of Radial Ball Bearings

Following are the various types of radial ball bearings:

1. ***Single row deep groove bearing.*** A single row deep groove bearing is shown in Fig. 27.3 (*a*).



(*a*) Single row deep groove. (*b*) Filling notch. (*c*) Angular contact. (*d*) Double row. (*e*) Self-aligning.

Fig. 27.3. Types of radial ball bearings.

During assembly of this bearing, the races are offset and the maximum number of balls are placed between the races. The races are then centred and the balls are symmetrically located by the use of a retainer or cage. The deep groove ball bearings are used due to their high load carrying capacity and suitability for high running speeds. The load carrying capacity of a ball bearing is related to the size and number of the balls.

Radial ball bearing

2. *Filling notch bearing.* A filling notch bearing is shown in Fig. 27.3 (*b*). These bearings have notches in the inner and outer races which permit more balls to be inserted than in a deep groove ball bearings. The notches do not extend to the bottom of the race way and therefore the balls inserted through the notches must be forced in position. Since this type of bearing contains larger number of balls than a corresponding unnotched one, therefore it has a larger bearing load capacity.

3. *Angular contact bearing.* An angular contact bearing is shown in Fig. 27.3 (*c*). These bearings have one side of the outer race cut away to permit the insertion of more balls than in a deep groove bearing but without having a notch cut into both races. This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load. The angular contact bearings are usually used in pairs so that thrust loads may be carried in either direction.

4. *Double row bearing.* A double row bearing is shown in Fig. 27.3 (*d*). These bearings may be made with radial or angular contact between the balls and races. The double row bearing is appreciably narrower than two single row bearings. The load capacity of such bearings is slightly less than twice that of a single row bearing.

5. *Self-aligning bearing.* A self-aligning bearing is shown in Fig. 27.3 (*e*). These bearings permit shaft deflections within 2-3 degrees. It may be noted that normal clearance in a ball bearing are too small to accommodate any appreciable misalignment of the shaft relative to the housing. If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur. Following are the two types of self-aligning bearings :

(*a*) Externally self-aligning bearing, and **(*b*)** Internally self-aligning bearing.

In an ***externally self-aligning bearing,*** the outside diameter of the outer race is ground to a spherical surface which fits in a mating spherical surface in a housing, as shown in Fig. 27.3 (*e*). In case of ***internally self-aligning bearing,*** the inner surface of the outer race is ground to a spherical

CHAPTER 26

Sliding Contact Bearings

26.1 Introduction

A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may be used.

26.2 Classification of Bearings

Though the bearings may be classified in many ways, yet the following are important from the subject point of view:

Roller Bearing

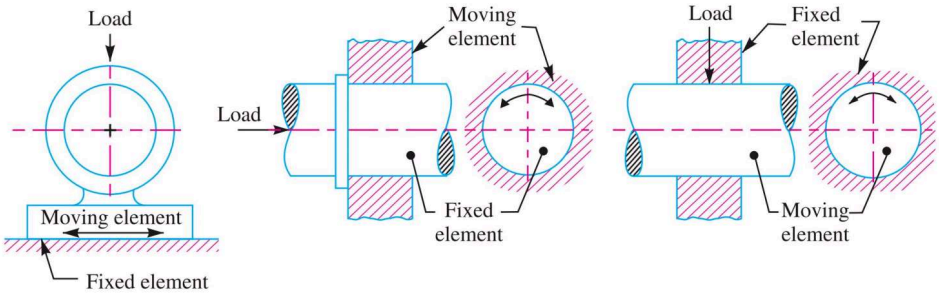
1. *Depending upon the direction of load to be supported.* The bearings under this group are classified as:

(*a*) Radial bearings, and **(*b*)** Thrust bearings.

In ***radial bearings***, the load acts perpendicular to the direction of motion of the moving element as shown in Fig. 26.1 (*a*) and (*b*).

In ***thrust bearings,*** the load acts along the axis of rotation as shown in Fig. 26.1 (*c*).

Note : These bearings may move in either of the directions as shown in Fig. 26.1.



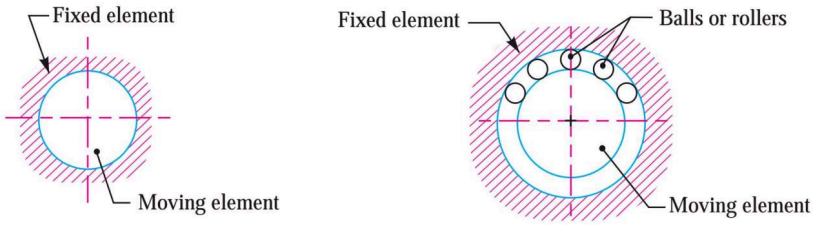
(*a*) Radial bearing. (*b*) Radial bearing. (*c*) Thrust bearing.

Fig. 26.1. Radial and thrust bearings.

2. *Depending upon the nature of contact.* The bearings under this group are classified as :

(*a*) Sliding contact bearings, and **(*b*)** Rolling contact bearings.

In ***sliding contact bearings,*** as shown in Fig. 26.2 (*a*), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as ***plain bearings.***



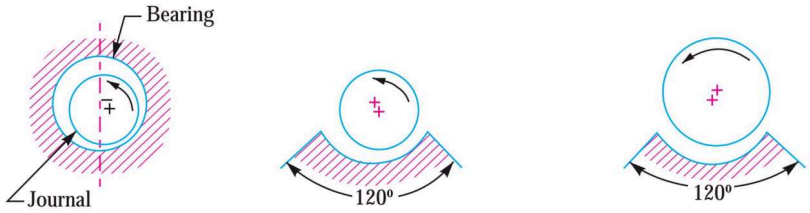
(*a*) Sliding contact bearing. (*b*) Rolling contact bearings.

Fig. 26.2. Sliding and rolling contact bearings.

In ***rolling contact bearings,*** as shown in Fig. 26.2 (*b*), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.

26.3 Types of Sliding Contact Bearings

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig. 26.1 (*a*), may be called ***slipper*** or ***guide bearings.*** Such type of bearings are usually found in cross-head of steam engines.



(*a*) Full journal bearing. (*b*) Partial journal bearing. (*c*) Fitted journal bearing.

Fig. 26.3. Journal or sleeve bearings.

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as ***journal*** or ***sleeve bearings.*** When the angle of contact of the bearing with the journal is 360° as shown in Fig. 26.3 (*a*), then the bearing is called a ***full journal bearing.*** This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.

When the angle of contact of the bearing with the journal is 120°, as shown in Fig. 26.3 (*b*), then the bearing is said to be ***partial journal bearing.*** This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as ***clearance bearings*** because the diameter of the journal is less than that of bearing.

Sliding contact bearings are used in steam engines

When a partial journal bearing has no clearance *i.e.* the diameters of the journal and bearing are equal, then the bearing is called a ***fitted bearing,*** as shown in Fig. 26.3 (*c*).

The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows :

1. ***Thick film bearings.*** The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as ***hydrodynamic lubricated bearings.***
2. ***Thin film bearings.*** The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other atleast part of the time. Such type of bearings are also called ***boundary lubricated bearings.***
3. ***Zero film bearings.*** The zero film bearings are those which operate without any lubricant present.
4. ***Hydrostatic or externally pressurized lubricated bearings.*** The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

26.4 Hydrodynamic Lubricated Bearings

Hydrodynamic Lubricated Bearings

We have already discussed that in hydrodynamic lubricated bearings, there is a thick film of lubricant between the journal and the bearing. A little consideration will show that when the bearing is supplied with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is eccentric with the bearing axis. The load can be supported by this fluid pressure without any actual contact between the journal and bearing. The load carrying ability of a hydrodynamic bearing arises simply because a viscous fluid resists being pushed around. Under the proper conditions, this resistance to motion will develop a pressure distribution in the lubricant film that can support a useful load. The load supporting pressure in hydrodynamic bearings arises from either

1. the flow of a viscous fluid in a converging channel (known as ***wedge film lubrication***), or
2. the resistance of a viscous fluid to being squeezed out from between approaching surfaces (known as ***squeeze film lubrication***).

26.5 Assumptions in Hydrodynamic Lubricated Bearings

The following are the basic assumptions used in the theory of hydrodynamic lubricated bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.
4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional, *i.e.* the side leakage is neglected.

26.6 Important Factors for the Formation of Thick Oil Film in Hydrodynamic Lubricated Bearings

According to Reynolds, the following factors are essential for the formation of a thick film of

5. Power transmission

Drives - Drives are used to transmit power from one shaft to another shaft at a considerable distance.

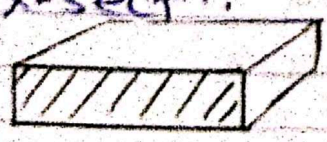
Types of drive -

1. Belt drive
2. Rope drive
3. Chain drive
4. Gear drive

1. Belt drive - Belt drive is used to transmit — " — by means of pulleys & belt. Belt drive is a ~~de~~ negative drive beoz of slip.

- Types of belt -

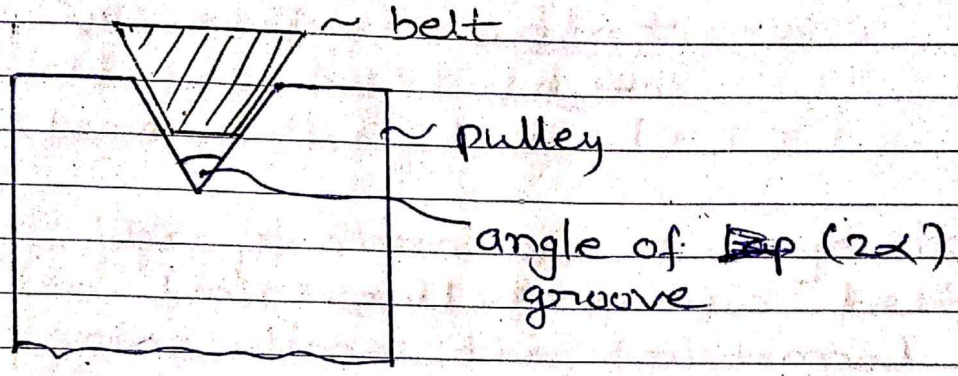
1. Flat belt - Flat belts is mostly used in factories & workshops where less amount of power is to be transmitted from one pulley to another. Centre distance b/w pulleys are not more than 10 m. They have rectangular x-sectn.



Flat belt.

V-belt

2. V- belt - The V- belt is mostly used in factories & workshopes where large amount of power is to be transmitted, when the pulleys are very near to each other.



Types of belt drive -

1. Open belt drive - Open belt drive is used ~~to~~ with shaft arranged in ||& rotating in same direction. The driver pulls the belt from lower side & delivers it to the upper side. So the tension on lower side will be more than on the upper side. The lower side is called tension side or tight side & upper side is called slack side.

2. Crossed belt drive - In case of crossed belt drive the shafts are || but ~~the~~ rotate in opposite direction.

then $VR = \frac{N_2}{N_1} = \frac{d_1}{d_2}$

when thickness of belt is considered then,

$$VR = \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

- Slip - The motion b/w pulleys is xmitted due to frictional grip b/w belt & pulley. But some times the frictional grip becomes less & due to this the driver rotates without carrying the belt with it, This may cause some forward motion of the belt without carrying the driven pulley with it. This is called slip & expressed as %.

- Due to slip vel. Ratio decreases.

$$V.R = \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right) \qquad (S = S_1 + S_2)$$

& $$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{S}{100}\right)$$

- S should be put in %age in que^n s.

- Length of open belt — खुला पट्टा

$$L_o = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

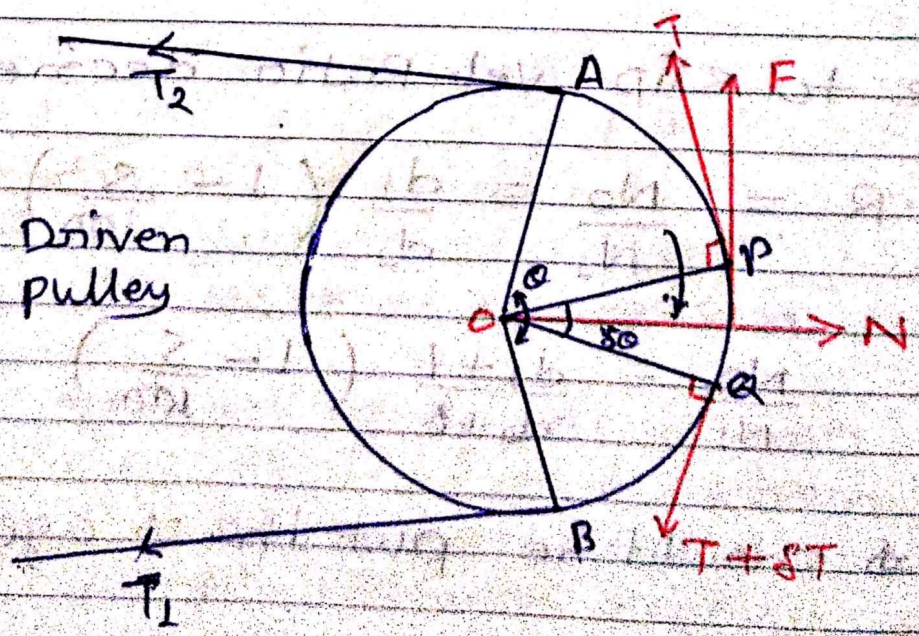
where - C = Centre distance b/w shaft
D = Dia of bigger shaft
d = Dia of smaller shaft

- Length of Cross belt - अयमुखी पट्टा

$$L_c = 2C + \frac{\pi(D+d)}{2} + \frac{(D+d)^2}{4C}$$

$$\sin\alpha = \frac{D-d}{2C}, \quad \theta = 180 \pm 2\alpha$$

- Ratio of tensions for flat belt -



Consider a driven pulley rotating in ACW direction.

Let $\delta\theta = \angle POQ$

μ = coeff. of friction b/w belt & pulley

N = normal reactⁿ

T_1 = Tension in tight side

T_2 = Tension in slack side

θ = angle of lap

Consider a small element PQ. ~~For equilibrium~~

The belt PQ is in equilibrium under the following forces -

1. Tension T in belt at P
2. Tension $(T+\delta T)$ in belt at Q
3. Normal reactⁿ N
4. Friction force $F = \mu N$

For equilibrium $\Sigma H = 0$

$$N - T\sin\frac{\delta\theta}{2} - (T+\delta T)\sin\frac{\delta\theta}{2} = 0$$

$\because$ $\delta\theta$ is very small,

$\therefore \sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$

$$\therefore N = T\cdot\frac{\delta\theta}{2} + (T+\delta T)\cdot\frac{\delta\theta}{2}$$

$$N = T\cdot\frac{\delta\theta}{2} + T\cdot\frac{\delta\theta}{2} + \cancel{\delta T\cdot\frac{\delta\theta}{2}}^{\,0}$$

$$N = T\cdot\delta\theta \quad \text{— (1)}$$

also $\Sigma V = 0$

$$F + T\cos\frac{\delta\theta}{2} - (T+\delta T)\cos\frac{\delta\theta}{2} = 0$$

$$F = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$$\mu N = \delta T \cos\frac{\delta\theta}{2}$$

$\because$ $\delta\theta$ is very small, $\therefore$ $\cos\frac{\delta\theta}{2} = 1$

$$\therefore \quad \mu N = \delta T$$

$$N = \frac{\delta T}{\mu} \quad \text{--- (2)}$$

equating (1) & (2)

$$T\delta\theta = \frac{\delta T}{\mu} \Rightarrow \frac{\delta T}{T} = \mu\,\delta\theta$$

Integrating both side ~~b/w~~ from A to B, b/w T_1 & T_2 & b/w 0 to θ

$$\therefore \int_{T_2}^{T_1} \frac{\delta T}{T} = \int_0^{\theta} \mu\,\delta\theta$$

$$(\ln T)_{T_2}^{T_1} = \mu[\theta]_0^{\theta}$$

$$\ln T_1 - \ln T_2 = \mu(\theta - 0)$$

$$\ln \frac{T_1}{T_2} = \mu\theta \Rightarrow \boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$ proved

Power transmitted by belt -

power = Workdone per second
= Force × distance
= $(T_1 - T_2) \times V$ ← velocity

$$\boxed{P = (T_1 - T_2)\, V.}$$ Watt

Effect of centrifugal tension on power -

Tension caused due to centrifugal force is called cent. tension. (T_c.)

Tension in tight side = $T_1 + T_c$
— " — slack " = $T_2 + T_c$

$\therefore$ power = $[(T_1 + T_c) - (T_2 + T_c)] \cdot V$

$$P = (T_1 - T_2) \cdot V$$

$\therefore$ No effect of T_c on power transmitted.

Max^m tension in belt = $(T_1 + T_c)$

- Condition of max^m power -

- $T_{max} = 3T_c$

- $P_{max} = (T_{max} \cdot v - mv^3) \cdot c$

where $c = \left[1 - \frac{1}{e^{\mu\theta}}\right]$

- For V-belt - $\frac{T_1}{T_2} = e^{\theta \cdot \mu \operatorname{cosec}\alpha}$

$\alpha =$

- For open belt $\theta = 180^\circ - 2\alpha$
  For cross belt $\theta = 180^\circ + 2\alpha$

- Advantages of V-belt over flat belt -

1. When distance b/w shaft is ~~more~~ less then V-belt is better than flat belt.

2. More power can be transmitted by V-belt.

3. More than one belts can be used for more power xmission.

4. There is very less slip for V-belt drive.

5. Shock absorbing capacity is also more than flat ~~pla~~ belt.

6. Less noise is generated.

Disadvantage -

1. For more centre distance, V-belt drive cannot be used.

2. Less reliable. (विश्वास)

3. Complex / difficult in construction.

• Rope drive -

Rope is used instead of belt when the distance b/w shaft is long & considerable power is to be transmitted. Here the rim of the pulley is grooved in which the rope runs.

The effect of groove is to increase the frictional grip of the rope on the pulley & to reduce the sliping.

Advantage over belt drive -

1. Used when the distance b/w shaft is large.
2. Frictional grip is more.
3. The net driving tension is more.

June 2016

5. $D = 450\ mm$, $d = 200\ mm$

$C = 1.95\ m$, cross belt

$L_c = ?$ θ_1 & $\theta_2 = ?$

$$L_c = 2C + \frac{\pi(D+d)}{2} + \frac{(D+d)^2}{4C}$$

$$= 2\times1.95 + \frac{\pi(0.45+0.2)}{2} + \frac{(0.45+0.2)^2}{4\times1.95}$$

$L_c = 4.975\ m$ Ans

Contact angle $\Rightarrow$ $\sin\alpha = \frac{D-d}{2C}$

$$\sin\alpha = \frac{0.45-0.2}{2\times1.95}$$

$\alpha = 3.675^\circ$

For cross belt $\theta = 180^\circ + 2\alpha$

$\theta = 187.35^\circ \times \frac{\pi}{180}$

$\theta = 3.27\ rad$ Ans

June - 2015

5(or). Solved.

Dec. 2014

5. $D = 600\ mm$, $N = 200\ rpm$

$\mu = 0.25$, $\theta = 180^\circ = 2.79\ rad$

$T_{max} = 2500\ N$

$$V = \frac{\pi d N}{60} = \frac{\pi \times 600 \times 200 \times 10^{-3}}{60}$$

$V = 6.28\ m/s$

Assume the condition of max^m power -

$$\therefore T_c = \frac{T_{max}}{3} = 833.33\ N$$

$$T_1 = T_{max} - T_c = 1666.67\ N$$

Now $\frac{T_1}{T_2} = e^{\mu\theta} \Rightarrow \frac{1666.67}{T_2} = e^{0.25 \times 2.79}$

$T_2 = 829.71\ N$

$P = (T_1 - T_2) V = 5.256\ kW$

Ans

June-2012

$d = 900\ mm$, $P = 20\ KW$

$N = 300\ rpm$, $\theta = 160° = 2.79\ rad$

$\mu = 0.25$, $\rho = 1\ gm/cm^3$

$t = 6\ mm$, $\sigma_{max} = 2\ N/mm^2$

$b_{min} = ?$

$$V = \frac{\pi d N}{60} = \frac{\pi \times 900 \times 300 \times 10^{-3}}{60}$$

$V = 14.13\ m/s$

$$P = (T_1 - T_2)\, V$$

$$20 \times 10^3 = (T_1 - T_2) \times 14.13$$

$$T_1 - T_2 = 1415.4 \quad \text{--- (1)}$$

& $$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times 2.79}$$

$$\frac{T_1}{T_2} = 2 \Rightarrow T_1 = 2T_2$$

Put value of T_1 in eq^n (1)

$$2T_2 - T_2 = 1415.4$$

$T_2 = 1415.4\ N$

& $T_1 = 2T_2 = 2830.8\ N$

also $T_{max} = \sigma_{max} \times A$

$T_{max} = 2 \times b \times 6 = 12b$

& $T_{max} = T_1 + T_c$

$12b = 2830.8 + 1.2b$

$T_c = mv^2 = \rho V \times v^2$

$= 1 \times 10^{-3} \times 10^{-3} \times 1000 \times b \times 6 \times 14.13^2$

(kg/mm^3)

$T_c = 1.2b$

$b = 262.11\ mm$ Ans

Example 11.11. *An open belt drive connects two pulleys 1.2 m and 0.5 m diameter, on parallel shafts 4 metres apart. The mass of the belt is 0.9 kg per metre length and the maximum tension is not to exceed 2000 N. The coefficient of friction is 0.3. The 1.2 m pulley, which is the driver, runs at 200 r.p.m. Due to belt slip on one of the pulleys, the velocity of the driven shaft is only 450 r.p.m. Calculate the torque on each of the two shafts, the power transmitted, and power lost in friction. What is the efficiency of the drive ?*

Solution. Given : $d_1 = 1.2$ m or $r_1 = 0.6$ m ; $d_2 = 0.5$ m or $r_2 = 0.25$ m ; $x = 4$ m ; $m = 0.9$ kg/m; $T = 2000$ N ; $\mu = 0.3$; $N_1 = 200$ r.p.m. ; $N_2 = 450$ r.p.m.

We know that velocity of the belt,

$$v = \frac{\pi d_1 . N_1}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.57 \text{ m/s}$$

and centrifugal tension, $T_C = m.v^2 = 0.9\ (12.57)^2 = 142$ N

∴ Tension in the tight side of the belt,

$$T_1 = T - T_C = 2000 - 142 = 1858 \text{ N}$$

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{0.6 - 0.25}{4} = 0.0875 \quad \text{or} \quad \alpha = 5.02°$$

∴ Angle of lap on the smaller pulley,

$$\theta = 180° - 2\alpha = 180° - 2 \times 5.02° = 169.96°$$

$$= 169.96 \times \pi / 180 = 2.967 \text{ rad}$$

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.967 = 0.8901$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8901}{2.3} = 0.387 \quad \text{or} \quad \frac{T_1}{T_2} = 2.438$$

...(Taking antilog of 0.387)

$$\therefore \quad T_2 = \frac{T_1}{2.438} = \frac{1858}{2.438} = 762 \text{ N}$$

Torque on the shaft of larger pulley

We know that torque on the shaft of larger pulley,

$$T_L = (T_1 - T_2)\, r_1 = (1858 - 762)\, 0.6 = 657.6 \text{ N-m} \quad \textbf{Ans.}$$

Torque on the shaft of smaller pulley

We know that torque on the shaft of smaller pulley,

$$T_S = (T_1 - T_2)\, r_2 = (1858 - 762)\, 0.25 = 274 \text{ N-m} \quad \textbf{Ans.}$$

Power transmitted

We know that the power transmitted,

$$P = (T_1 - T_2)\, v = (1858 - 762)\, 12.57 = 13780 \text{ W}$$

$$= 13.78 \text{ kW} \quad \textbf{Ans.}$$

Power lost in friction

We know that input power,

$$P_1 = \frac{T_L \times 2\pi N_1}{60} = \frac{657.6 \times 2\pi \times 200}{60} = 13\,780 \text{ W} = 13.78 \text{ kW}$$

and output power,

$$P_2 = \frac{T_S \times 2\pi N_2}{60} = \frac{274 \times 2\pi \times 450}{60} = 12\,910 \text{ W} = 12.91 \text{ kW}$$

∴ Power lost in friction $= P_1 - P_2 = 13.78 - 12.91 = 0.87$ kW **Ans.**

Efficiency of the drive

We know that efficiency of the drive,

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{12.91}{13.78} = 0.937 \text{ or } 93.7\% \quad \textbf{Ans.}$$

Derive - $V = \sqrt{\frac{T_{max}}{3m}}$

Sol^n ⇒ We know that, power transmitted by belt.

$$P = (T_1 - T_2) \times V \quad \text{— (1)}$$

where T_1 = Tension in tight side (in N)
T_2 = Tension in slack side (in N)
V = velocity of belt (in m/s)

also, $\frac{T_1}{T_2} = e^{\mu\theta}$

$$\Rightarrow \quad T_2 = \frac{T_1}{e^{\mu\theta}}$$

putting the value of T_2 in eq^n (1), we get

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right) \times V$$

$$P = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right) \times V$$

Let $c = 1 - \frac{1}{e^{\mu\theta}}$

then,

$$P = T_1 \cdot c \cdot V \quad \text{— (2)}$$

Now, if T = max^m tension in belt, then

$$T = T_1 + T_c$$ {where T_c = Centrifugal tension}

$$\therefore T_1 = T - T_c$$

Substituting the value in eq^n (2), we get

$$P = (T - T_c) \cdot c \cdot V$$
$$= (T - mv^2) \cdot c \cdot V$$

$$P = (TV - mV^3) \cdot c \quad \text{— (3)}$$

Now, for max^m power $\frac{dP}{dV} = 0$

$$\therefore \frac{d}{dV}(TV - mV^3) \cdot c = 0$$

$$T - 3mV^2 = 0$$

$$T = 3mV^2 \Rightarrow V^2 = \frac{T}{3m}$$

$$\boxed{V = \sqrt{\frac{T}{3m}}}$$

proved

Formulae for Numericals -

Belt drive -

1. Length of Open belt -

$$L_o = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

2. Length of cross belt -

$$L_c = 2C + \frac{\pi(D+d)}{2} + \frac{(D+d)^2}{4C}$$

where C = Centre distance b/w shafts
D = diameter of bigger pulley
d = diameter of smaller pulley.

3. For angle of contact -

$$\sin\alpha = \frac{D-d}{2C} \Rightarrow \text{find } \alpha$$

then: For open belt $\theta = 180^\circ - 2\alpha$

For cross belt $\theta = 180^\circ + 2\alpha$

4. For flat belt - $\frac{T_1}{T_2} = e^{\mu\theta}$

T_1 = tension in tight side
T_2 = tension in slack side
μ = Co-efficient of friction b/w belt & pulley
θ = Contact angle / Lap angle (in radians)

5. Power transmitted by belt -

$$P = (T_1 - T_2) \times V \qquad \left\{ V = \frac{\pi DN}{60} \right\}$$

6. When centrifugal tension (T_c) taken into account then,

Tension in tight side $= T_1 + T_c$

Tension in slack side $= T_2 + T_c$

max^m tension in belt - $T_{max} = T_1 + T_c$

7. For max^m power -

$$T_c = \frac{T_{max}}{3}$$

& $$v = \sqrt{\frac{T_{max}}{3m}}$$

8. mass m = density x area x length

$$m = \rho \times b \times t \times L$$

9. $$T_c = m.v^2$$

10. Initial tension in belt - (T_0) then.

$$T_0 = \frac{T_1 + T_2}{2}$$

when centrifugal tension is considered.

$$T_0 = \frac{T_1 + T_2 + 2T_c}{2}$$

11. For V-belt - $$\frac{T_1}{T_2} = e^{\theta . \mu \operatorname{cosec} \alpha}$$

Ap. May. 2017 - $D = 600\ mm$

$N = 200\ rpm$

$\mu = 0.25$

$\theta = 160^\circ = 2.8\ radian$

$T_{max} = 2500\ N = T_1$

For flat belt. $\frac{T_1}{T_2} = e^{\mu\theta}$

$$\frac{2500}{T_2} = e^{0.25 \times 2.8}$$

$$T_2 = 1241.5\ N$$

& velocity $V = \frac{\pi DN}{60}$

$$= \frac{\pi \times 600 \times 200 \times 10^{-3}}{60}$$

$$V = 6.28\ m/s$$

∴ power $P = (T_1 - T_2) \times V$

$$P = (2500 - 1241.5) \times 6.28$$

$$P = 7.9\ KW$$ Ans

A.M. 2018 given,

$D = 450\ mm$, $d = 200\ mm$

$C = 1.95\ m$

cross belt $L_c = ?$, $\theta = ?$

We know that, Length of cross belt

$$L_c = 2C + \frac{\pi(D+d)}{2} + \frac{(D+d)^2}{4C}$$

$$L_c = 2 \times 1.95 + \frac{\pi (0.45 + 0.2)}{2} + \frac{(0.45 + 0.2)^2}{4 \times 1.95}$$

$$L_c = 4.97 \text{ m}$$ Ans

Now, $\sin\alpha = \frac{D-d}{2C} = \frac{0.45 - 0.2}{2 \times 1.95}$

$$\sin\alpha = 0.064$$

$$\therefore \quad \alpha = 3.67^\circ$$

Angle of Contact

$$\therefore \quad \theta = 180^\circ + 2\alpha$$

$$= 180^\circ + 2 \times 3.67^\circ$$

$$\theta = 187.34^\circ$$ Ans

"OR"

Differences between flat belt drive and V-belt drive	
Flat Belt Drive	**V-Belt Drive**
1. Flat belts have a rectangular cross-section, where width of belt is much larger than thickness.	1. V-belts have a trapezoidal cross-section, where maximum width is almost same with the thickness.
2. The pulleys for flat belt drive is cylindrical type. These are simple in construction and thus cheaper.	2. Pulleys must contain V-slot in the periphery having slot angles compatible with that of the belt. Thus these pulleys are costlier.
3. Only one surface of the belt remains in contact with the pulley.	3. Two inclined surfaces continuously remain in contact with the pulley.
4. Flat belts can be used for transmitting power to a long center distance (even up to 15m).	4. V-belt drive is suitable for power transmission in short center distance (usually below 1m).
5. It can be used either in open belt configuration or in crossed belt configuration.	5. V-belt is always used in open belt configuration.
6. Slip between the belt and pulley occurs frequently in flat belt drive.	6. Due to higher friction force for wedge action, slip reduces, especially in low speed.
7. Flat belts are jointed (hinged), so their operation is noisy.	7. V-belts are endless, so their operation is quite.

Advantages and Disadvantages of Chain Drive over Belt or Rope Drive

Advantages

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.
2. Since the chains are made of metal, therefore they occupy less space in width than a belt or rope drive.
3. It may be used for both long as well as short distances.
4. It gives a high transmission efficiency (upto 98 percent).
5. It gives less load on the shafts.
6. It has the ability to transmit motion to several shafts by one chain only.
7. It transmits more power than belts.
8. It permits high speed ratio of 8 to 10 in one step.
9. It can be operated under adverse temperature and atmospheric conditions.

Disadvantages

1. The production cost of chains is relatively high.

2. The chain drive needs accurate mounting and careful maintenance, particularly lubrication and slack adjustment.

3. The chain drive has velocity fluctuations especially when unduly stretched.

Comparison of belt ,chain and gear drive:-

particulars	Belt drive	Chain drive	Gear drive
➢ Main elements	Pulleys, belt	sprockets ,chain	Gear
➢ Slip	Slip may occurs	No slip (Positive drive)	No slip (positive dive)
➢ Suitability	For large centre distance	For moderate centre distance	For short centre distance
➢Space requires	large	Moderate	less
➢ Design	Simplest	Simple	Complicated
➢ Failure	Failure of belts does not cause the further damage of machine	Failure of chain may not seriously damage the machine	Failure of gear may cause serious break down in the machine
➢ Life	Less	Moderate	Long
➢Lubrication	Not required	Require	Require proper
➢Installation cost	Less	Moderate	More
➢ Use	For low VR	For moderate VR	For high VR
➢ e.g.	Use as a first drive in transmission	Bicycle , Automobile	gear boxes , automobile

12.3. Advantages and Disadvantages of Gear Drive

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

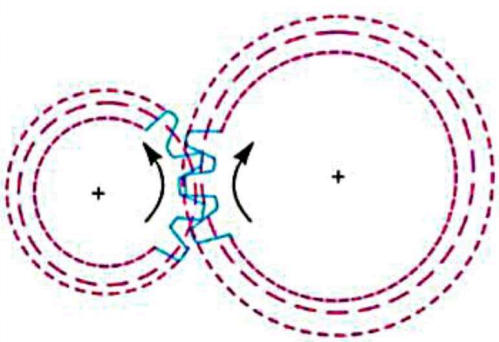
Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

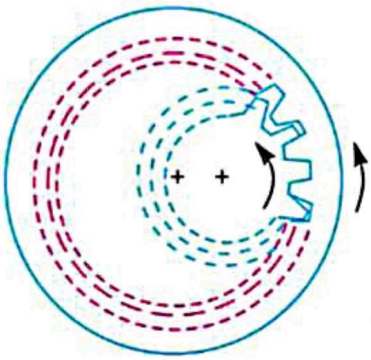
3. *According to the type of gearing*. The gears, according to the type of gearing may be classified as :

(*a*) External gearing, **(*b*)** Internal gearing, and **(*c*)** Rack and pinion.

In ***external gearing***, the gears of the two shafts mesh externally with each other as shown in Fig. 12.3 (*a*). The larger of these two wheels is called ***spur wheel*** and the smaller wheel is called **pinion.** In an external gearing, the motion of the two wheels is always ***unlike***, as shown in Fig. 12.3 (*a*).



(*a*) External gearing.



(*b*) Internal gearing.

Fig. 12.3

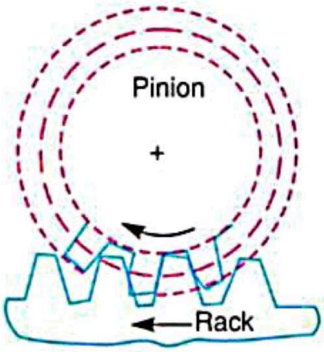


Fig. 12.4. Rack and pinion.

In ***internal gearing***, the gears of the two shafts mesh ***internally*** with each other as shown in Fig. 12.3 (*b*). The larger of these two wheels is called ***annular wheel*** and the smaller wheel is called ***pinion***. In an internal gearing, the motion of the two wheels is always ***like***, as shown in Fig. 12.3 (*b*).

4. *According to position of teeth on the gear surface*. The teeth on the gear surface may be (*a*) straight, (*b*) inclined, and (*c*) curved.

We have discussed earlier that the spur gears have straight teeth where as helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

Internal gears

Rack and pinion

12.5. Terms Used in Gears

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig. 12.5.

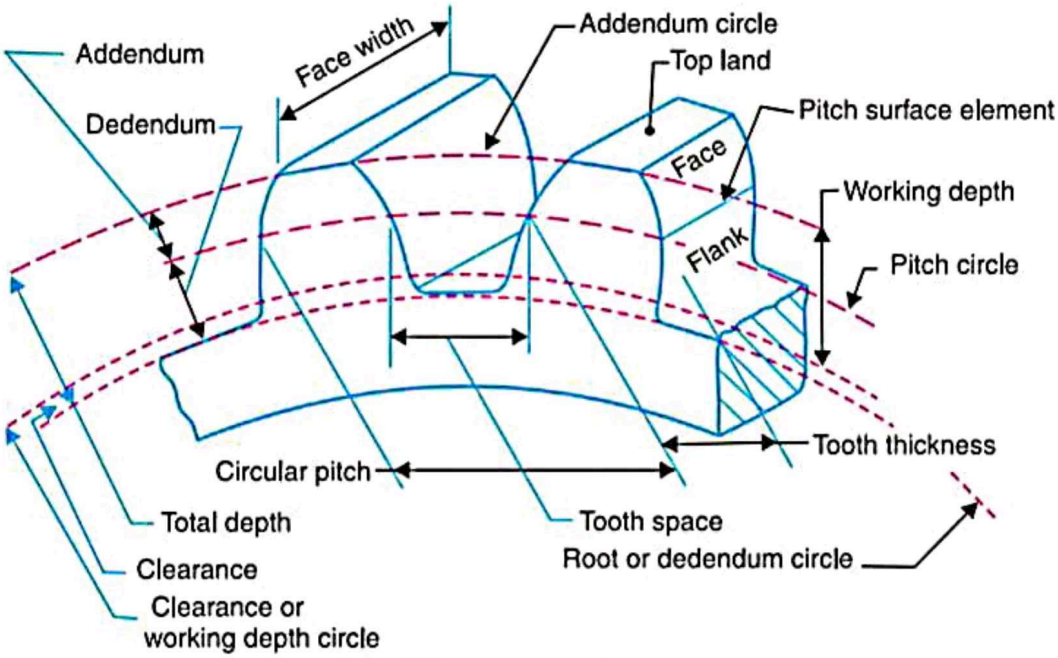


Fig. 12.5. Terms used in gears.

1. *Pitch circle*. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

* A straight line may also be defined as a wheel of infinite radius.

2. *Pitch circle diameter*. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.

3. *Pitch point*. It is a common point of contact between two pitch circles.

4. *Pitch surface*. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. *Pressure angle or angle of obliquity*. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are $14\frac{1}{2}°$ and 20°.

6. *Addendum*. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. *Dedendum*. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. *Addendum circle*. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. *Dedendum circle*. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter × cos ϕ, where ϕ is the pressure angle.

10. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

11. *Diametral pitch*. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d. Mathematically,

Diametral pitch, $p_d = \frac{T}{D} = \frac{\pi}{p_c}$ $\quad \ldots\left(\because p_c = \frac{\pi D}{T}\right)$

where T = Number of teeth, and

D = Pitch circle diameter.

12. *Module*. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically,

Module, $m = D/T$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

13. *Clearance*. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle***.

14. *Total depth*. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

15. *Working depth*. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. *Tooth thickness*. It is the width of the tooth measured along the pitch circle.

17. *Tooth space*. It is the width of space between the two adjacent teeth measured along the pitch circle.

18. *Backlash*. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. *Face of tooth*. It is the surface of the gear tooth above the pitch surface.

20. *Flank of tooth*. It is the surface of the gear tooth below the pitch surface.

21. *Top land*. It is the surface of the top of the tooth.

22. *Face width*. It is the width of the gear tooth measured parallel to its axis.

23. *Profile*. It is the curve formed by the face and flank of the tooth.

24. *Fillet radius*. It is the radius that connects the root circle to the profile of the tooth.

25. *Path of contact*. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. ****Length of the path of contact*. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. *****Arc of contact*. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

(*a*) ***Arc of approach***. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) ***Arc of recess***. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note : The ratio of the length of arc of contact to the circular pitch is known as ***contact ratio*** *i.e.* number of pairs of teeth in contact.

12.6. Gear Materials

12.16. Length of Path of Contact

Consider a pinion driving the wheel as shown in Fig. 12.11. When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at K (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and* ends at L (outer end of the tooth face on the pinion or on the flank near the base circle of wheel). MN is the common normal at the point of contacts and the common tangent to the base circles. The point K is the intersection of the addendum circle of wheel and the common tangent. The point L is the intersection of the addendum circle of pinion and common tangent.

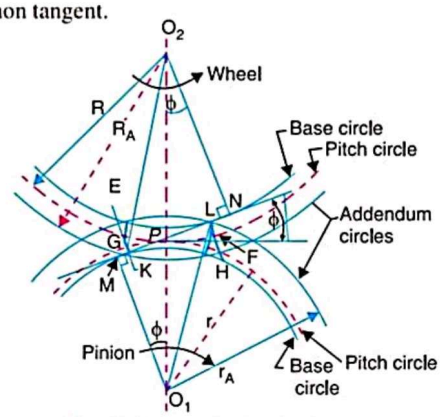
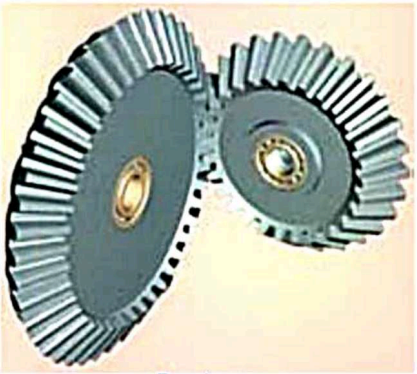


Fig. 12.11. Length of path of contact.

* If the wheel is made to act as a driver and the directions of motion are reversed, then the contact between a pair of teeth begins at L and ends at K.

We have discussed in Art. 12.4 that the length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion. Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as ***path of approach*** and the part of the path of contact PL is known as ***path of recess.***



Bevel gear

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P$ = Radius of pitch circle of wheel.

From Fig. 12.11, we find that radius of the base circle of pinion,

$$O_1M = O_1P \cos\phi = r\cos\phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P \cos\phi = R\cos\phi$$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

and $$PN = O_2P\sin\phi = R\sin\phi$$

∴ Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly from right angled triangle O_1ML,

and $$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

∴ Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

∴ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

12.17. Length of Arc of Contact

We have already defined that the arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Fig. 12.11, the arc of contact is *EPF* or *GPH*. Considering the arc of contact *GPH*, it is divided into two parts *i.e.* arc *GP* and arc *PH*. The arc *GP* is known as ***arc of approach*** and the arc *PH* is called ***arc of recess***. The angles subtended by these arcs at O_1 are called ***angle of approach*** and ***angle of recess*** respectively.

We know that the length of the arc of approach (arc *GP*)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

and the length of the arc of recess (arc *PH*)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

Since the length of the arc of contact *GPH* is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact

$$= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

12.18. Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the **ratio of the length of the arc of contact to the circular pitch.** Mathematically,

Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{p_c}$$

where p_c = Circular pitch = πm, and

m = Module.

Notes : 1. The contact ratio, usually, is not a whole number. For example, if the contact ratio is 1.6, it does not mean that there are 1.6 pairs of teeth in contact. It means that there are alternately one pair and two pairs of teeth in contact and on a time basis the average is 1.6.

2. The theoretical minimum value for the contact ratio is one, that is there must always be at least one pair of teeth in contact for continuous action.

3. Larger the contact ratio, more quietly the gears will operate.

2. *Differential gear of an automobile.* The differential gear used in the rear drive of an automobile is shown in Fig. 13.21. Its function is

(*a*) to transmit motion from the engine shaft to the rear driving wheels, and

(*b*) to rotate the rear wheels at different speeds while the automobile is taking a turn.

As long as the automobile is running on a straight path, the rear wheels are driven directly by the engine and speed of both the wheels is same. But when the automobile is taking a turn, the outer wheel will run faster than the * inner wheel because at that time the outer rear wheel has to cover more distance than the inner rear wheel. This is achieved by epicyclic gear train with bevel gears as shown in Fig. 13.21.

The bevel gear *A* (known as pinion) is keyed to the propeller shaft driven from the engine shaft through universal coupling. This gear *A* drives the gear *B* (known as crown gear) which rotates freely on the axle *P*. Two equal gears *C* and *D* are mounted on two separate parts *P* and *Q* of the rear axles respectively. These gears, in turn, mesh with equal pinions *E* and *F* which can rotate freely on the spindle provided on the arm attached to gear *B*.

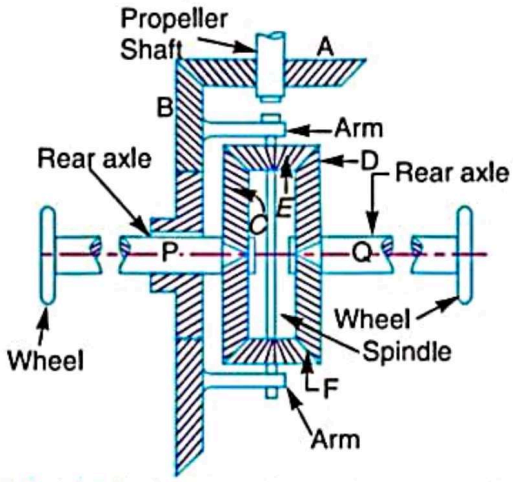


Fig. 13.21. Differential gear of an automobile.

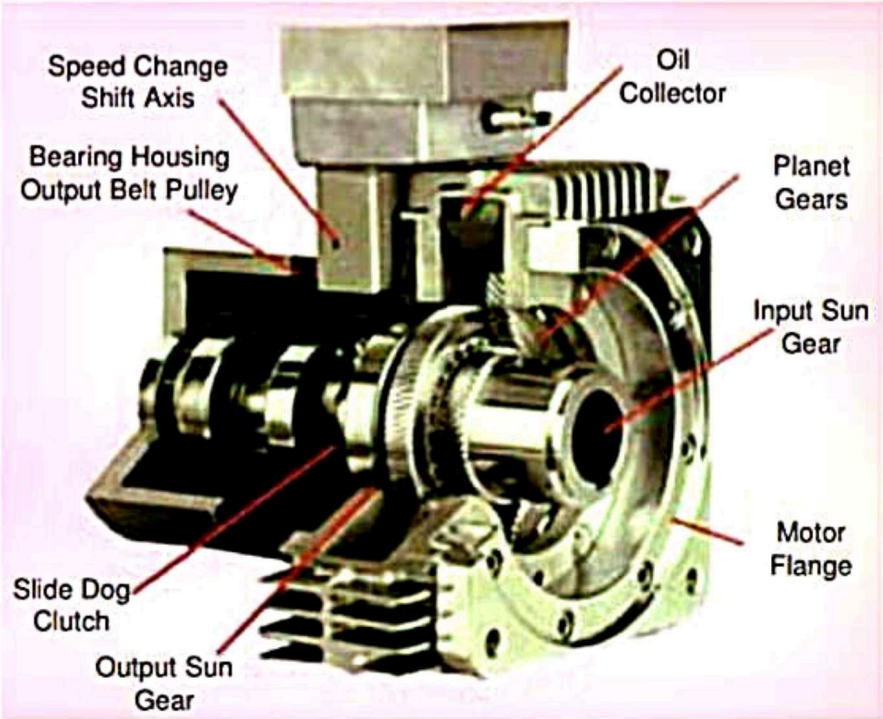
When the automobile runs on a straight path, the gears *C* and *D* must rotate together. These gears are rotated through the spindle on the gear *B*. The gears *E* and *F* do not rotate on the spindle. But when the automobile is taking a turn, the inner rear wheel should have lesser speed than the outer rear wheel and due to relative speed of the inner and outer gears *D* and *C*, the gears *E* and *F* start rotating about the spindle axis and at the same time revolve about the axle axis.

Due to this epicyclic effect, the speed of the inner rear wheel decreases by a certain amount and the speed of the outer rear wheel increases, by the same amount. This may be well understood by drawing the table of motions as follows :

* This difficulty does not arise with the front wheels as they are greatly used for steering purposes and are mounted on separate axles and can run freely at different speeds.

13.9. Compound Epicyclic Gear Train—Sun and Planet Gear

A compound epicyclic gear train is shown in Fig. 13.9. It consists of two co-axial shafts S_1 and S_2, an annulus gear A which is fixed, the compound gear (or planet gear) B-C, the sun gear D and the arm H. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm H. The sun gear is co-axial with the annulus gear and the arm but independent of them.



Sun and Planet gears.

The annulus gear A meshes with the gear B and the sun gear D meshes with the gear C. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

Note : The gear at the centre is called the ***sun gear*** and the gears whose axes move are called ***planet gears.***

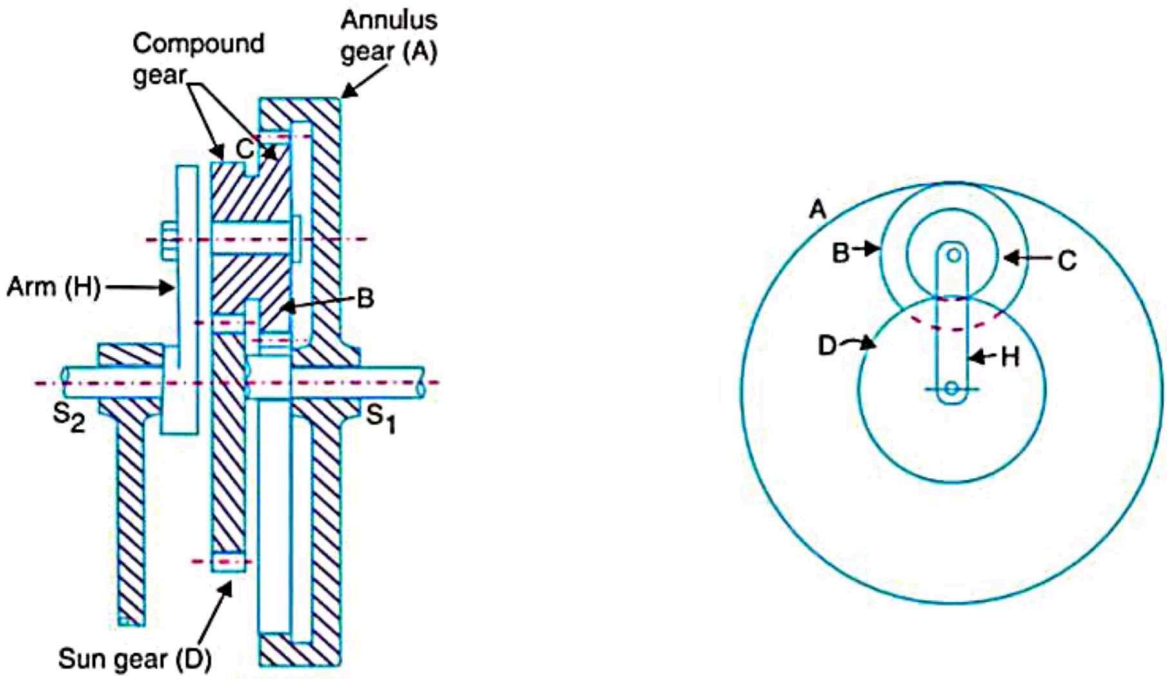


Fig. 13.9. Compound epicyclic gear train.

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given - $t = 20$, $T = 60$, $\phi = 20°$

$m = 8\,mm$, $A_P = A_G = 10\,mm$

$G = \frac{T}{t} = 3$

Now, pitch circle radius of pinion & gear -

$$r = \frac{mt}{2} = \frac{8 \times 20}{2} = 80 \text{ mm}$$

$$R = \frac{mT}{2} = \frac{8 \times 60}{2} = 240 \text{ mm}$$

& Addendum circle radius of pinion & gear -

$$r_A = r + \text{Addendum} = r + A_P$$

$$r_A = 80 + 10 = 90 \text{ mm}$$

& $R_A = R + A_G = 240 + 10 = 250$ mm

Now, max^m permissible radius of addendum circle upto which no interference occurs.

$$(r_A)_{max} = \sqrt{(r_b)^2 + [(r+R)\sin\phi]^2} \quad \left\{ \begin{array}{l} r_b = r\cos\phi \\ r_b = 75.2 \text{ mm} \end{array} \right.$$

$$= \sqrt{(75.2)^2 + [(80+240)\sin 20°]^2}$$

$$(r_A)_{max} = 132.8 \text{ mm}$$

& $(R_A)_{max} = \sqrt{(R_b)^2 + [(r+R)\sin\phi]^2}$ $\left\{ R_b = R\cos\phi = 225.5 \text{ mm} \right.$

$= \sqrt{(225.5)^2 + (320 \sin 20^\circ)^2}$

$(R_A)_{max} = 250.65$ mm

$\because$ $R_A < (R_A)_{max}$ & $r_A < (r_A)_{max}$

means the addendum circle radius of both pinion & gear is less than the upper limit. Thus no interference occurs.

(2) Length of path of approach -

$= \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$

$KP = \sqrt{(250)^2 - (240\cos 20^\circ)^2} - 240\sin 20^\circ$

$KP = 25.8$ mm Ans

Length of path of recess -

$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$= \sqrt{90^2 - (80\cos 20^\circ)^2} - 80\sin 20^\circ$

$PL = 22.1$ mm

(3) Arc of approach $= \dfrac{\text{Length of path of approach}}{\cos\phi}$

$= 27.45$ mm Ans

$$\text{Arc of recess} = \frac{\text{Length of path of recess}}{\cos\phi}$$

$$= 23.5 \text{ mm}$$

(4) Length of path of contact -

$$= KP + PL$$

$$= 25.8 + 22.1$$

$$= 47.9 \text{ mm}$$

$$\text{Arc of contact} = \frac{\text{Length of path of contact}}{\cos\phi}$$

$$= \frac{47.9}{\cos 20^\circ}$$

$$= 51 \text{ mm}$$

Ans

Flywheel -

* For Numericals -

1. Maximum fluctuation of energy -

$$(\Delta E)_{max} = E_{max} - E_{min}.$$

$$(\Delta E)_{max} = \text{max. energy} - \text{min}^m \text{ energy}$$

2. Co-efficient of fluctuation of energy -

It is the ratio of max^m fluctuation of energy to the workdone per cycle.

$$C_E = \frac{(\Delta E)_{max}}{W_{cycle}}$$

3. Workdone per cycle -

$$W_{cycle} = T_{mean} \times \theta$$

T_{mean} = mean torque / resisting torque

θ = angle turned in one revolution
[must be in radians ; eg - $\pi, 2\pi, 3\pi$]

$\theta = 2\pi$ in case of steam engine & 2-stroke engine.

$\theta = 4\pi$ in case of 4-stroke IC engine

4. Power - $P = \frac{2\pi N T_{mean}}{60}$ Watts

N = speed in rpm.
T_{mean} = mean torque in N-m

5. Work done per cycle - (other method)

$$W_{cycle} = \frac{Power \times 60}{n}$$

$n = N$ for steam engine & 2-stroke ICE.
$= \frac{N}{2}$ for 4-stroke IC engine.

6. Coefficient of fluctuation of speed -

$$C_s = \frac{N_1 - N_2}{N} = \frac{\omega_1 - \omega_2}{\omega}$$

N_1 = max^m speed ; N = mean speed
N_2 = min^m speed ; $N = \frac{N_1 + N_2}{2}$
ω = speed in rad/s

7. Co-efficient of steadiness -

$$m = \frac{1}{C_s} = \frac{N}{N_1 - N_2} = \frac{\omega}{\omega_1 - \omega_2}$$

8. Energy stored in flywheel - (K.E.)

$$E = \frac{1}{2} I.\omega^2 = \frac{1}{2} \cdot mk^2 . \omega^2$$

where m = mass of flywheel (kg)

I = mass moment of inertia (kg-m^2)
$I = mk^2$

k = radius of gyration

For ring - $K = R$
For Disc - $K = R/\sqrt{2}$

9. Max^m fluctuation of energy -

$$(\Delta E)_{max} = E_{max} - E_{min}$$

$$= \frac{1}{2} I \omega_{max}^2 - \frac{1}{2} I \omega_{min}^2$$

$$= \frac{1}{2} I (\omega_{max} + \omega_{min}) \cdot (\omega_{max} - \omega_{min})$$

$$= \frac{1}{\cancel{2}} I \frac{(\omega_{max} + \omega_{min})}{2} \frac{(\omega_{max} - \omega_{min})}{\omega} \times \cancel{2}\omega$$

$$= I C_s \cdot \omega^2$$

* $(\Delta E)_{max} = I \omega^2 C_s$

Note - 1. If C_s is given as 2%, then $C_s = 0.02$
if C_s is $\pm$ 2%, then $C_s = 0.04$

2. Any positive or -ve area above or below Tmean line is called fluctuation of energy.

3. At Tmean line $\Rightarrow$ sum of all +ve area = sum of all -ve area

4. Basic function of flywheel - stores & delivers energies at desired speed.

5. For high speed engines - I of flywheel should be less.

$\sin\theta \rightarrow 2\pi$

$\cos\theta \rightarrow 2\pi$

$\tan\theta \rightarrow \pi$

$\sin 3\theta \rightarrow \frac{2\pi}{3}$

$\cos 5\theta \rightarrow \frac{2\pi}{5}$

$f(\theta) = 100 + 200\sin 3\theta + 300\cos 5\theta + 100\tan 7\theta$

$\frac{2\pi}{3}, \frac{2\pi}{5}, \frac{\pi}{7}$ – Lcm

$= \frac{\text{Lcm of } N^r}{\text{HCF of } D^r} = \frac{2\pi}{1}$

TOM

Governors

The main function of governor is to maintain the engine speed within the prescribed limits for varying load condit^n.

Thus the ~~governor~~ Flywheel is a device which controls the speed variations caused by the fluctuation in turning moment. The flywheel does not control the speed variation caused by a varying load.

Thus Governor is a device which automatically controls the supply of fuel to the engine with the varying load condition.

- Governors
  - Centrifugal Gov.
    - Pendulum Gov. (Watt Gov.)
    - Loaded Gov.
      - Dead weight type
        - Porter Gov.
        - Proell Gov.
      - Spring loaded Gov.
        - Hartnell Gov.
        - Hartung Gov.
        - Pickering Gov.
  - Inertia Gov.

Difference b/w Governor & flywheel

Governor - 1. The function of the Gov. is to keep the speed ~~of~~ constant due to variation of load over a no. of cycles.

2. Governor controls change of speed.

3. It is automatic control device.

4. Governor regulate speed by regulating fuel.

5. Governor takes care of change of quality or quantity of working fluid.

5. This is an essential element of every engine.

Flywheel : 1. The function of flyweel is to decrease the variation in speed due to variation in turning moment in a cycle.

2. Flywheel controls rate of change of speed.

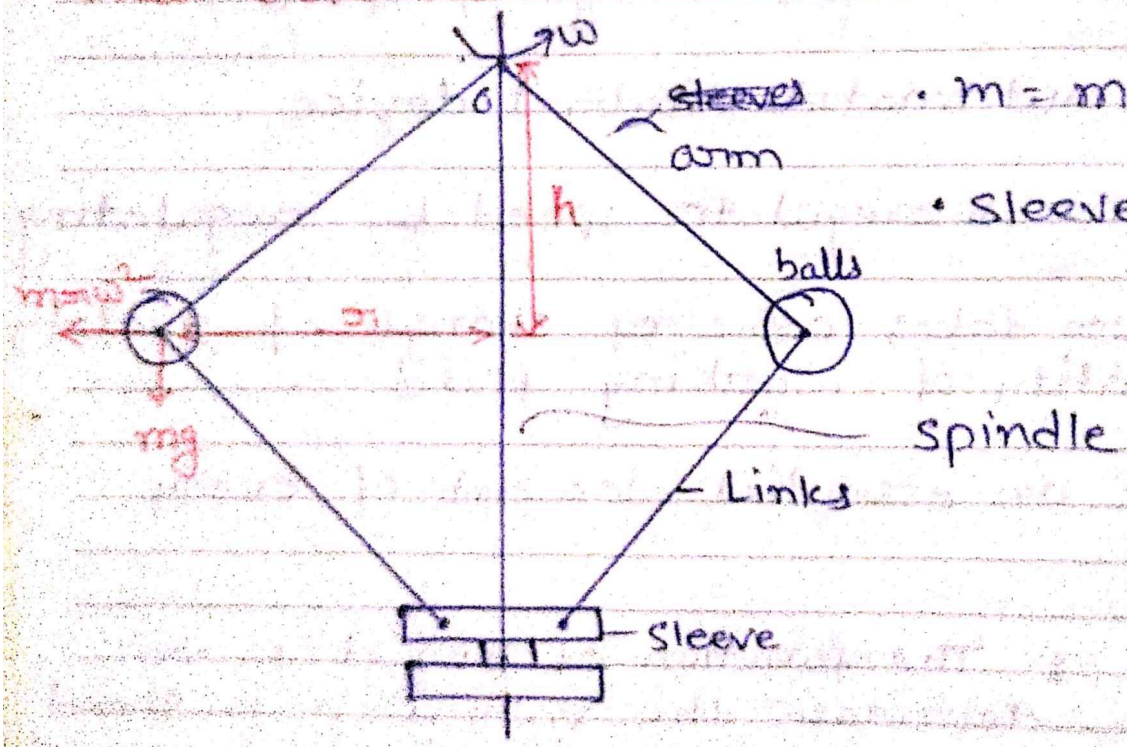
3. It is semi-automatic device.

4. Flywheel stores energy & gives it when required.

5. No control on working fluid's quality or quantity.

6. This is not an essential for all engines.

Watt Governor =



- m = mass of flyball
- Sleeve is massless

Taking moment about point O

$$(m r \omega^2) \cdot h = mg \cdot r$$

$$\omega^2 = \frac{g}{h} \Rightarrow \left(\frac{2\pi}{60}\right)^2 N^2 = \frac{g}{h}$$

$$\boxed{N^2 = \frac{895}{h}}$$

$\{g = 9.81\ m/s^2$

Watt Governor is simplest type of Centrifugal governor (sir james watt). It is basically a conical pendulum with links connecting flyballs & sleeve of negligible mass.

Let m = mass of each ball in kg
W = weight = mg in Newtons
T = Tension in the arms
ω = angular velocity of balls about spindle axis (rad/s)

r = radius of rotation of balls
i.e. horizontal distance of ball from spindle axis (m)

h = height of governor (m)

F_c = Centrifugal force ~~on~~ acting on balls
$= mr\omega^2$ (N)

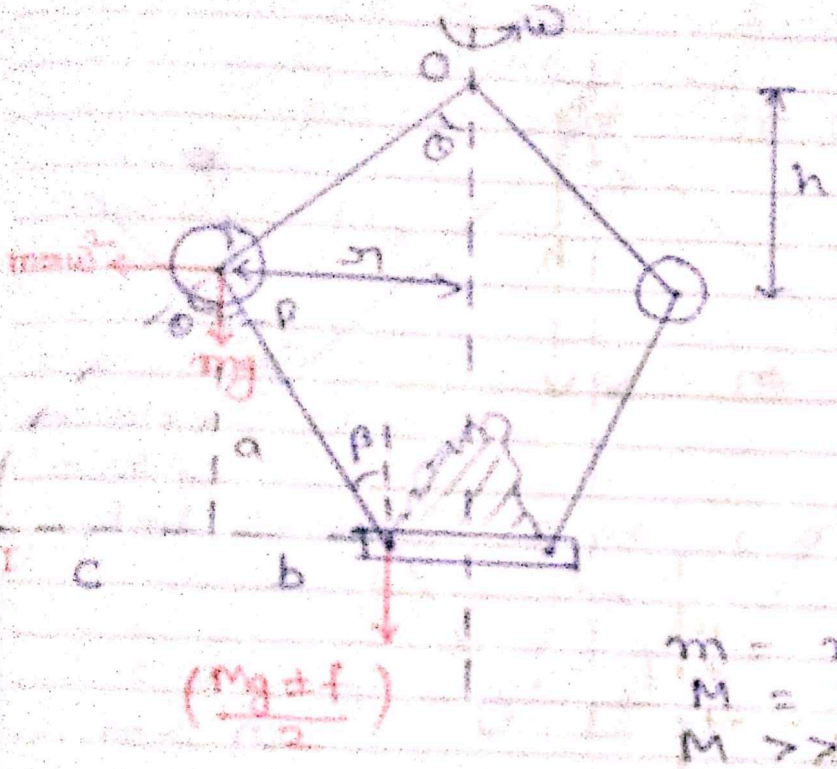
In Watt governor two arms are connected in spindle's top edge. In the other ends of arms a ball is attached (flyball). This ends are also connected with ~~a~~ link which is further connected to sleeve. The sleeve can move up & down (reciprocate) on the spindle. A bell crank lever is attached with sleeve whose other end

is connected to a throttle valve. The function of throttle valve is to regulate fuel supply.

The spindle of governor is connected with engine shaft by a bevel gear or belt drive; so that spindle cam rotates with rotation of engine shaft. For the prespecified speed of the shaft / spindle, the balls rotates ~~at~~ about ~~their~~ axis at constant radius & thus the sleeve remains at their specific position & the throttle valve continues the supply of fuel to engine.

But when the load on engine increase the speed of engine reduces & sleeves of governor moves down. Due to this the throttle valve opens & ~~to~~ increase the fuel supply to increase the speed of engine at increased load.

Porter Governor -



Porter Gov. is similar to Watt Gov. with a dead weight W on sleeve & it moves along with the sleeve on the spindle axis up & down.

m = mass of flyball
M = Mass of sleeve
M >>> m

Taking Moment about I -

$$(m r \omega^2) \cdot a = mg \cdot c + \frac{(Mg \pm f)}{2}(b+c)$$

$$N^2 = \frac{895}{h}\left[\frac{2mg + (Mg \pm f)(1+K)}{2mg}\right]$$

where $K = \frac{\tan\beta}{\tan\theta}$

If $f = 0$

$$N^2 = \frac{895}{h}\left[\frac{2mg + Mg(1+K)}{2mg}\right]$$

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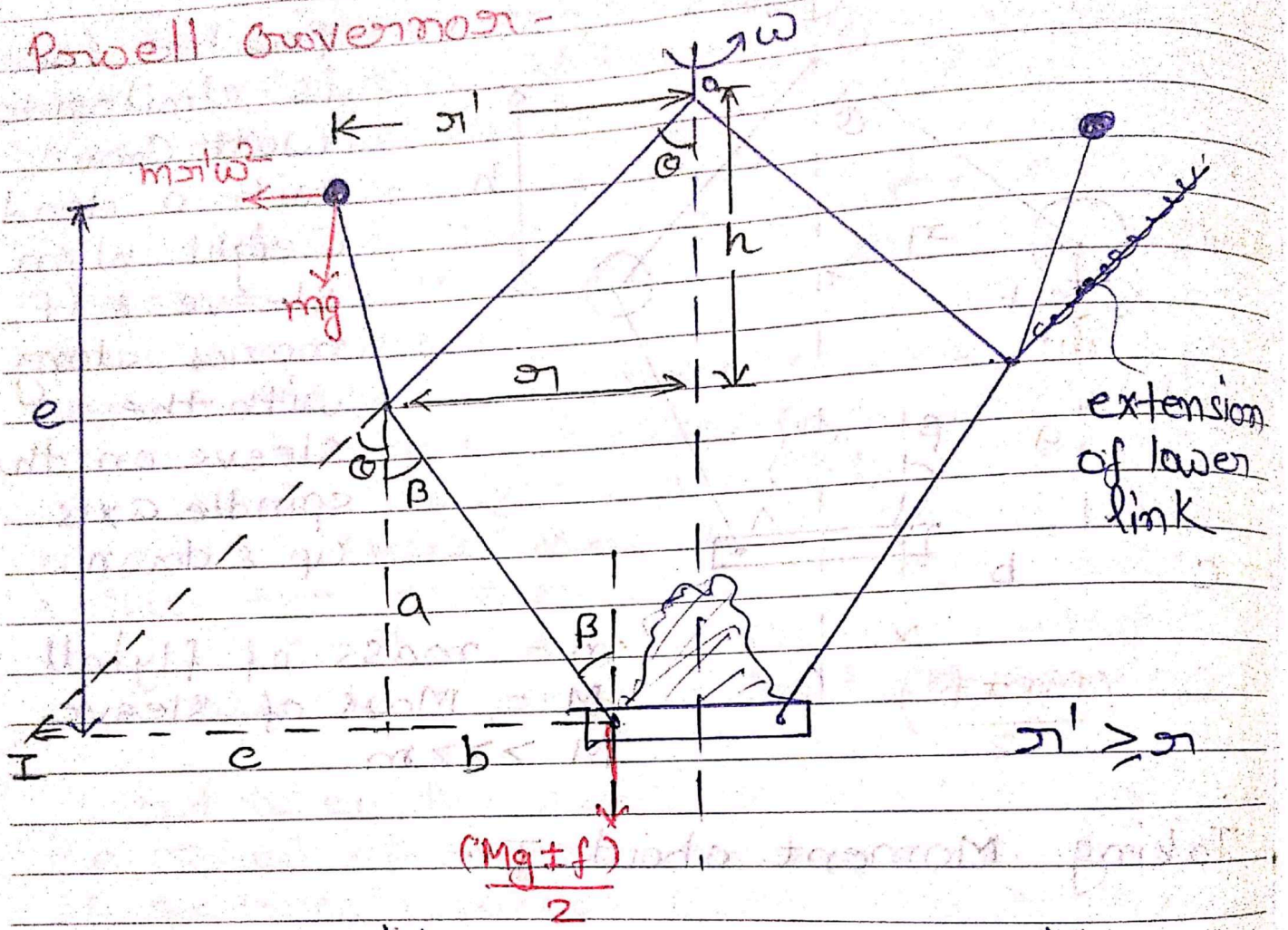
Proell Governor -



only one position where the extension of LL is ∥ to the gov. axis - $r' = r$

It is similar to Porter governor but the only difference is that the balls are attached at the extension of Lower link

The working principle is similar to watt Gov. or porter gov.

The instantaneous centre I lies at

the point of intersection of extension of upper arm & a horizontal st. line from the sleeve.

Taking moment about I

$$(mr'\omega^2) \cdot e = mg\,(c + r - r') + \frac{(Mg \pm f)}{2}(b+c)$$

Note - When the extension of LL is ∥ to Governor axis, i.e. $r = r'$ then

$$(mr'\omega^2) \cdot e = mg \cdot c + \frac{(Mg \pm f)}{2}(b+c)$$

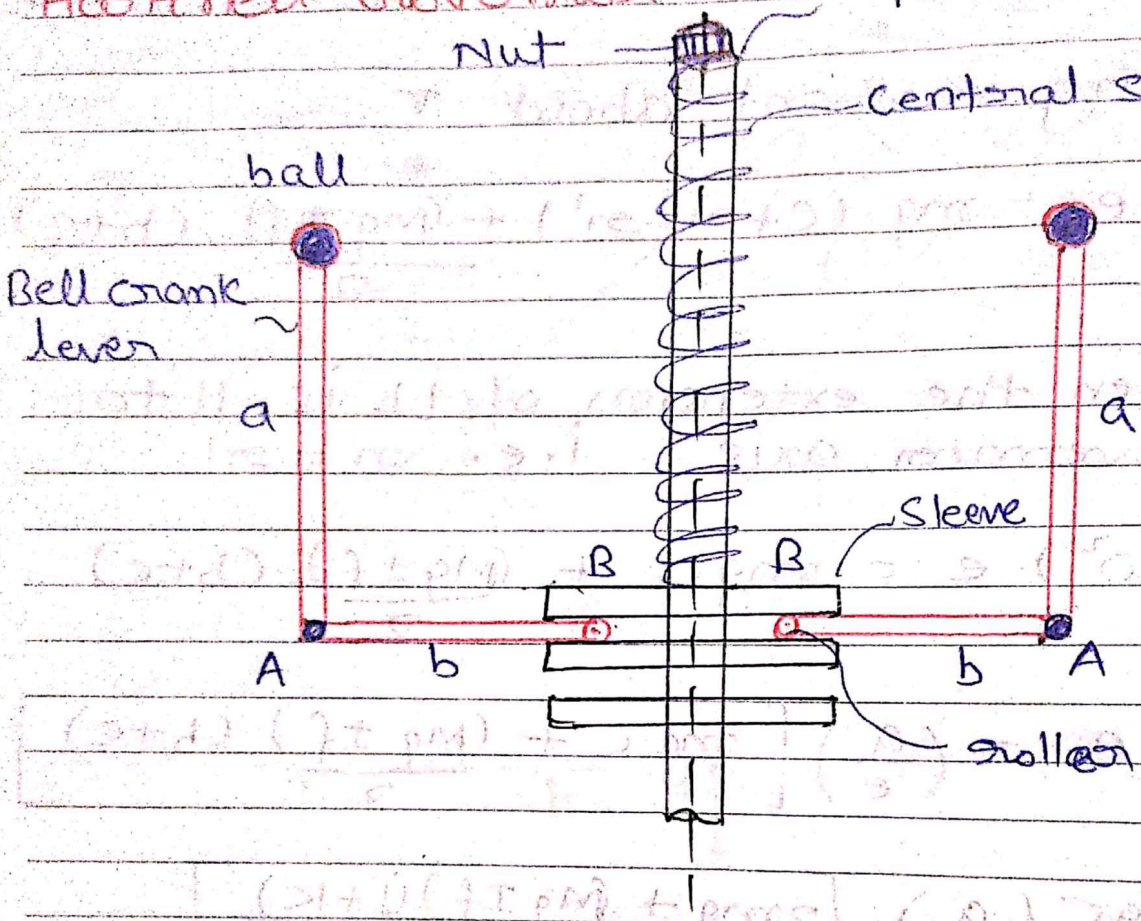
$$(mr'\omega^2) \cdot a = \left(\frac{a}{e}\right)\left[mg \cdot c + \frac{(Mg \pm f)}{2}(b+c)\right]$$

$$N^2 = \frac{895}{h}\left(\frac{a}{e}\right)\left[\frac{2mg \pm (Mg \pm f)(1+K)}{2mg}\right]$$

Note - If sleeve of governor displaced up

$r \uparrow$ } - $\omega \uparrow$ - stability
$r' \uparrow$ } - $\omega \downarrow$ - unstability
- ω = Constant = Isochronous

Hartnell Governor -



• Stiffness =

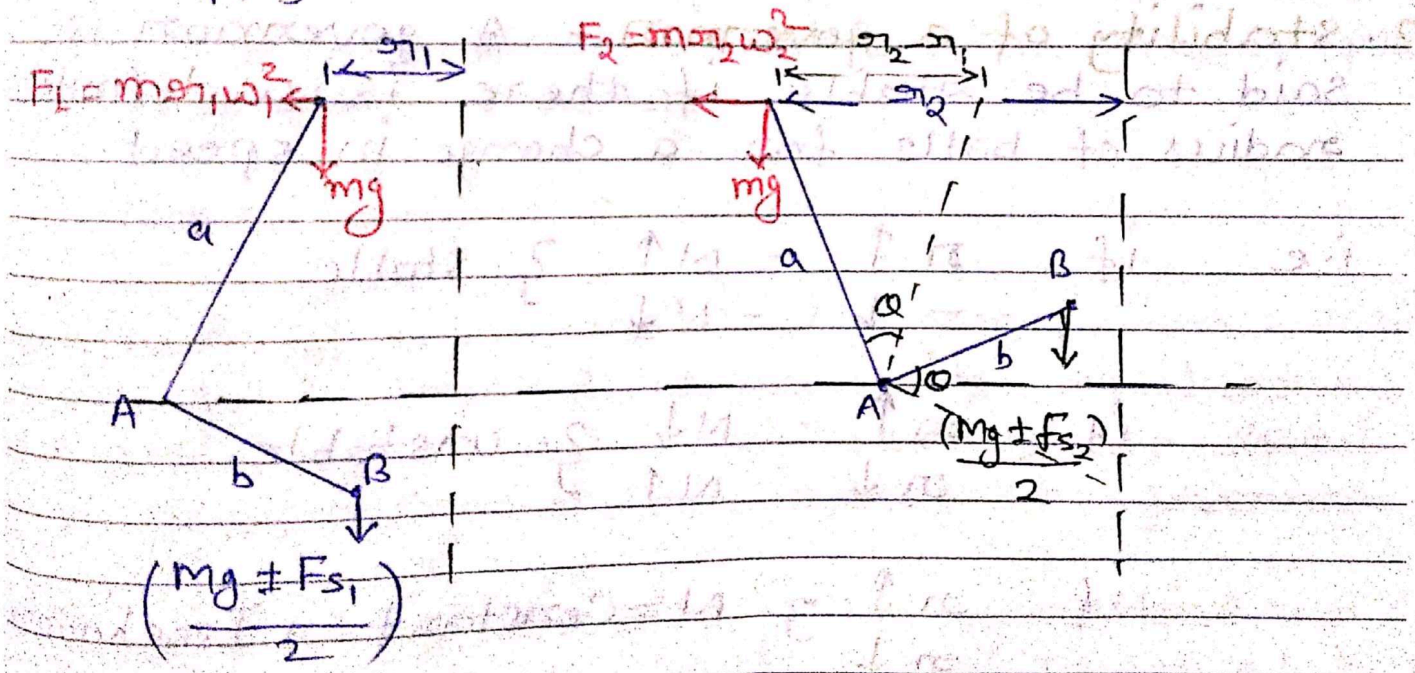
• Always in Compressi

It consists of two bell crank lever pivoted to the frame which is attached to the spindle & rotates with it. Balls are fixed at one end & roller at other end.

Let m = mass of ball
M = Mass of sleeve
r_1 & r_2 = min^m & max^m radius
ω_1 & ω_2 = min^m & max^m speed
F_1 & F_2 = Centrifugal forces
F_{s_1} & F_{s_2} = Spring forces on sleeve corresponding to min^m & max^m radii

The sleeve can move up & down on spindle. The sleeve is pressed by a spring, whose other end is ~~at~~ locked by nut at the top of spindle.

When spindle rotates, the balls also rotates with it & due to centrifugal force balls rotates at a bigger circle & due to which the sleeve moves ~~up~~ up & this displacement is transfered to throttle valve which controls & regulates the fuel ~~so~~ supply.



Total sleeve movement $= b\theta = \frac{b}{a}(r_2 - r_1)$

Addition compression in spring $= (x_2 - x_1) = \frac{b}{a}(r_2 - r_1)$

$(F_{s_2} - F_{s_1}) = (F_2 - F_1)\frac{2a}{b} = \frac{b}{a}(r_2 - r_1) \cdot S$

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Definitions -

1. Sensitivity of Governor - The movement of sleeve for a change in speed of Gov. is called sensitivity of governor.

It is the ratio of mean speed of gov. to the difference b/w max^m & min^m speed of governor.

$$\text{Sensitivity} = \frac{N}{N_1 - N_2} \quad ; \quad N = \frac{N_1 + N_2}{2}$$

2. Stability of a governor - A governor is said to be stable if there is a definite radius of balls for a change in speed.

i.e if $r \uparrow$ - $N \uparrow$ } stable
$r \downarrow$ - $N \downarrow$

if $r \uparrow$ - $N \downarrow$ } unstable
$r \downarrow$ - $N \uparrow$

if $r \uparrow$ } N = Constant ⇒ Isochronous
$r \downarrow$

It is highly desirable property.

$$\text{Sensitivity} \propto \frac{1}{\text{Stability}}$$

3. Isochronous Governor - A gov. is said to be isochronous if its equilibrium speed is constant for all radii of rotation.

$r \uparrow$ } N = Constant
$r \downarrow$

Sensitivity = ∞ ; $(\text{Hunting})_{Iso} = 0$

4. Hunting of Governor - An excessively fast to & fro motion of sleeve b/w stoppers is known as hunting.

If the speed of engine fluctuates continuously above & below the mean speed it is called hunting.

Sensitivity = 0

5. Effort of Governor - The mean force acting on the sleeve to change its equilibrium position for the %age change in speed of gov. is known as effort of governor.

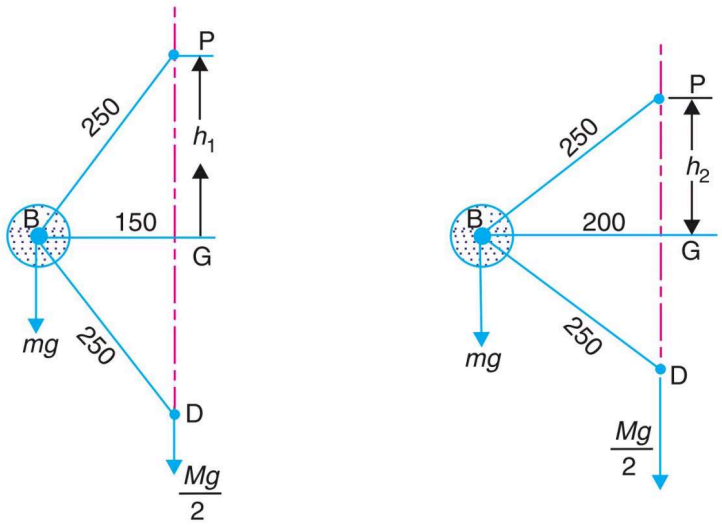
6. Power of governor - The workdone on the sleeve — " — is called power of gov.

mean effort = $\frac{E}{2}$; Power = mean effort x sleeve movement

Power $P = \frac{E}{2} \times$ Sleeve movement.

Example 18.2. *A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.*

Solution. Given : $BP = BD = 250$ mm $= 0.25$ m ; $m = 5$ kg ; $M = 15$ kg ; $r_1 = 150$ mm $= 0.15$m; $r_2 = 200$ mm $= 0.2$ m



(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.5

The minimum and maximum positions of the governor are shown in Fig. 18.5 (*a*) and (*b*) respectively.

Minimum speed when $r_1 = BG = 0.15$ m

Let N_1 = Minimum speed.

From Fig. 18.5 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.15)^2} = 0.2 \text{ m}$$

We know that

$$(N_1)^2 = \frac{m + M}{m} \times \frac{895}{h_1} = \frac{5 + 15}{5} \times \frac{895}{0.2} = 17\ 900$$

$\therefore$ $N_1 = 133.8$ r.p.m. **Ans.**

Maximum speed when $r_2 = BG = 0.2$ m

Let N_2 = Maximum speed.

From Fig. 18.5 (*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.2)^2} = 0.15 \text{ m}$$

We know that

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+15}{5} \times \frac{895}{0.15} = 23\ 867$$

$\therefore$ $N_2 = 154.5$ r.p.m. **Ans.**

Range of speed

We know that range of speed

$$= N_2 - N_1 = 154.4 - 133.8 = 20.7 \text{ r.p.m. } \textbf{Ans.}$$

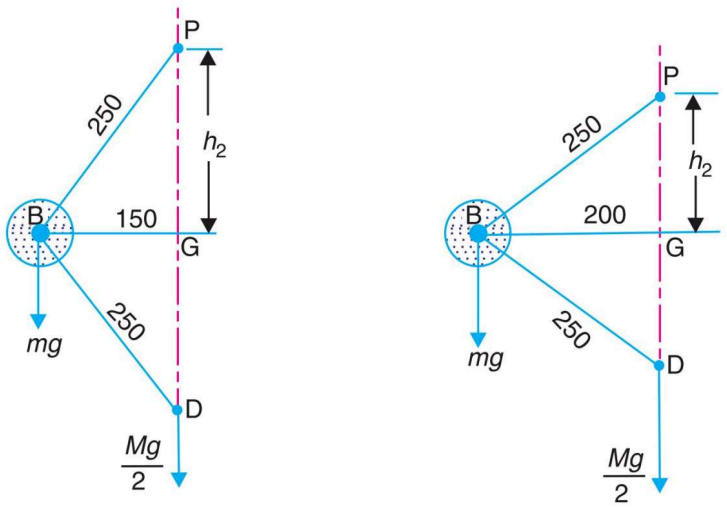
Example 18.3. *The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.*

Solution. Given : $BP = BD = 250$ mm ; $m = 5$ kg ; $M = 30$ kg ; $r_1 = 150$ mm ; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 18.6 (*a*) and (*b*) respectively.

Let N_1 = Minimum speed when $r_1 = BG = 150$ mm, and

N_2 = Maximum speed when $r_2 = BG = 200$ mm.



(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.6

Speed range of the governor

From Fig. 18.6 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

We know that

$$(N_1)^2 = \frac{m + M}{m} \times \frac{895}{h_1} = \frac{5 + 30}{5} \times \frac{895}{0.2} = 31\,325$$

$\therefore$ $N_1 = 177$ r.p.m.

From Fig. 18.6 (*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

We know that

$$(N_2)^2 = \frac{m + M}{m} \times \frac{895}{h_2} = \frac{5 + 30}{5} \times \frac{895}{0.15} = 41\,767$$

$\therefore$ $N_2 = 204.4$ r.p.m.

We know that speed range of the governor

$$= N_2 - N_1 = 204.4 - 177 = 27.4 \text{ r.p.m. } \textbf{Ans.}$$

Speed range when friction at the sleeve is equivalent of 20 N of load (i.e. when F = 20 N)

We know that when the sleeve moves downwards, the friction force (F) acts upwards and the minimum speed is given by

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 - 20)}{5 \times 9.81} \times \frac{895}{0.2} = 29\,500$$

$$\therefore \qquad N_1 = 172 \text{ r.p.m.}$$

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 + 20)}{5 \times 9.81} \times \frac{895}{0.15} = 44\,200$$

$$\therefore \qquad N_2 = 210 \text{ r.p.m.}$$

A series of hydel generators.

Note : This picture is given as additional information and is not a direct example of the current chapter.

We know that speed range of the governor

$$= N_2 - N_1 = 210 - 172 = 38 \text{ r.p.m. } \textbf{Ans.}$$

Example 18.4. *In an engine governor of the Porter type, the upper and lower arms are 200 mm and 250 mm respectively and pivoted on the axis of rotation. The mass of the central load is 15 kg, the mass of each ball is 2 kg and friction of the sleeve together with the resistance of the operating gear is equal to a load of 25 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° and 40°, find, taking friction into account, range of speed of the governor.*

Solution . Given : $BP = 200$ mm $= 0.2$ m ; $BD = 250$ mm $= 0.25$ m ; $M = 15$ kg ; $m = 2$ kg ; $F = 25$ N ; $\alpha_1 = 30°$; $\alpha_2 = 40°$

First of all, let us find the minimum and maximum speed of the governor.

The minimum and maximum position of the governor is shown Fig. 18.7 (*a*) and (*b*) respectively.

Let $\qquad N_1 =$ Minimum speed, and

$\qquad N_2 =$ Maximum speed.

From Fig. 18.7 (*a*), we find that minimum radius of rotation,

$$r_1 = BG = BP \sin 30° = 0.2 \times 0.5 = 0.1 \text{ m}$$

Height of the governor,

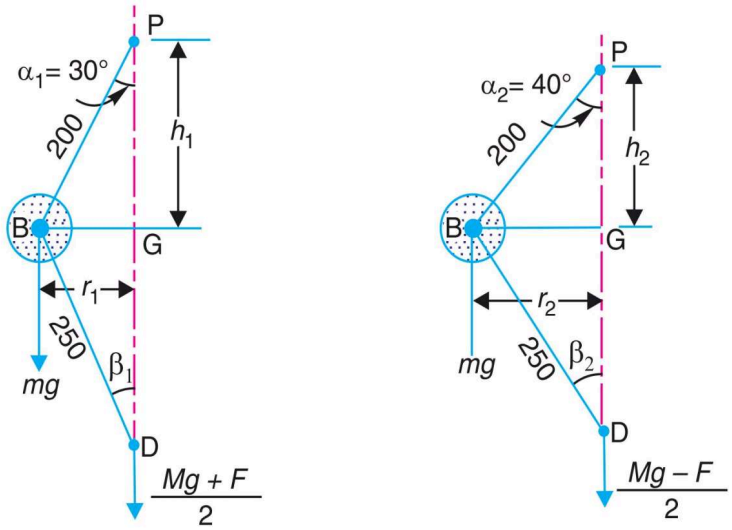
$$h_1 = PG = BP \cos 30° = 0.2 \times 0.866 = 0.1732 \text{ m}$$

and $$DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1)^2} = 0.23 \text{ m}$$

$\therefore$ $$\tan \beta_1 = BG/DG = 0.1/0.23 = 0.4348$$

and $$\tan \alpha_1 = \tan 30° = 0.5774$$

$\therefore$ $$q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.4348}{0.5774} = 0.753$$



All dimensions in mm.

(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.7

We know that when the sleeve moves downwards, the frictional force (F) acts upwards and the minimum speed is given by

$$(N_1)^2 = \frac{m.g + \left(\frac{M.g - F}{2}\right)(1 + q_1)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{2 \times 9.81 + \left(\frac{15 \times 9.81 - 24}{2}\right)(1 + 0.753)}{2 \times 9.81} \times \frac{895}{0.1732} = 33596$$

$\therefore$ $$N_1 = 183.3 \text{ r.p.m.}$$

Now from Fig. 18.7 (*b*),we find that maximum radius of rotation,

$$r_2 = BG = BP \sin 40° = 0.2 \times 0.643 = 0.1268 \text{ m}$$

Height of the governor,

$$h_2 = PG = BP \cos 40° = 0.2 \times 0.766 = 0.1532 \text{ m}$$

and $$DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1268)^2} = 0.2154 \text{ m}$$

$\therefore$ $$\tan \beta_2 = BG/DG = 0.1268 / 0.2154 = 0.59$$

and $$\tan \alpha_2 = \tan 40° = 0.839$$

$\therefore$ $$q_2 = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{0.59}{0.839} = 0.703$$

We know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by

$$(N_2)^2 = \frac{m.g + \left(\frac{m.g + F}{2}\right)(1 + q_2)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{2 \times 9.81 + \left(\frac{15 \times 9.81 + 24}{2}\right)(1 + 0.703)}{2 \times 9.81} \times \frac{895}{0.1532} = 49\ 236$$

$\therefore$ $N_2 = 222$ r.p.m.

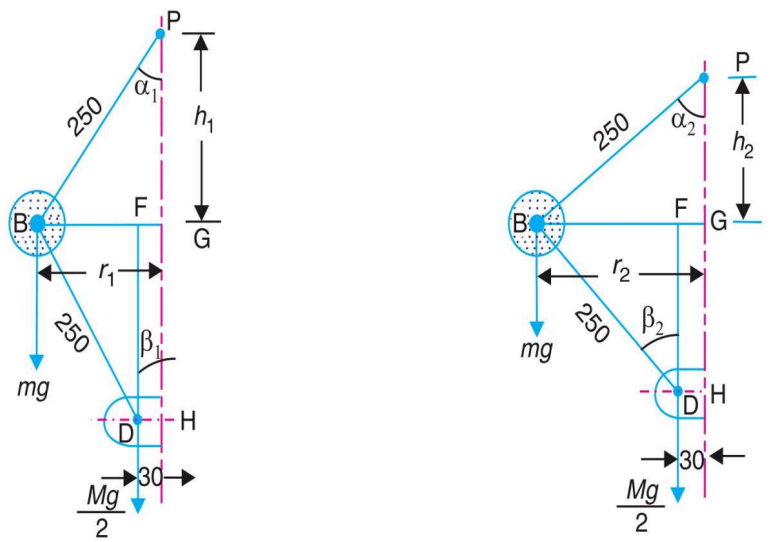
We know that range of speed

$= N_2 - N_1 = 222 - 183.3 = 38.7$ r.p.m. **Ans.**

Example 18.5. *A Porter governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg and the sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm and 200 mm. Determine the range of speed of the governor.*

Solution. Given : $BP = BD = 250$ mm ; $DH = 30$ mm ; $m = 5$ kg ; $M = 50$ kg ; $r_1 = 150$ mm ; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 18.8 (*a*) and (*b*) respectively.



(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.8

Let N_1 = Minimum speed when $r_1 = BG = 150$ mm ; and

N_2 = Maximum speed when $r_2 = BG = 200$ mm.

From Fig. 18.8 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

$$BF = BG - FG = 150 - 30 = 120 \text{ mm} \qquad \ldots (\because FG = DH)$$

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (120)^2} = 219 \text{ mm}$$

$\therefore$ $\tan \alpha_1 = BG/PG = 150 / 200 = 0.75$

and $\tan \beta_1 = BF/DF = 120/219 = 0.548$

$\therefore$ $$q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.548}{0.75} = 0.731$$

We know that $$(N_1)^2 = \frac{m + \frac{M}{2}(1 + q_1)}{m} \times \frac{895}{h_1} = \frac{5 + \frac{50}{2}(1 + 0.731)}{5} \times \frac{895}{0.2} = 43\,206$$

$\therefore$ $N_1 = 208$ r.p.m.

From Fig. 18.8(*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

$$BF = BG - FG = 200 - 30 = 170 \text{ mm}$$

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (170)^2} = 183 \text{ mm}$$

$\therefore$ $\tan \alpha_2 = BG/PG = 200/150 = 1.333$

and $\tan \beta_2 = BF/DF = 170/183 = 0.93$

$\therefore$ $$q_2 = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{0.93}{1.333} = 0.7$$

We know that

$$(N_2)^2 = \frac{m + \frac{M}{2}(1 + q_2)}{m} \times \frac{895}{h_2} = \frac{5 + \frac{50}{2}(1 + 0.7)}{5} \times \frac{895}{0.15} = 56\,683$$

$\therefore$ $N_2 = 238$ r.p.m.

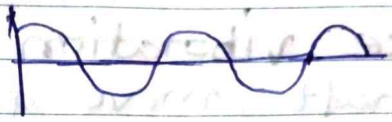
We know that range of speed

$= N_2 - N_1 = 238 - 208 = 30$ r.p.m. **Ans.**

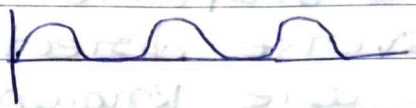
10. Vibration

- **Vibration** - To & fro periodic motion about their equilibrium / mean / zero position.

eg - Sin wave 

cos wave 

} vibrat^n

$\sin^2$ wave  No vibration

- Types of vibration.

1. Based on Force acting -
(a) Natural / free vibration
(b) Forced vibration
(c) ~~Dam~~ Forced damped vibration

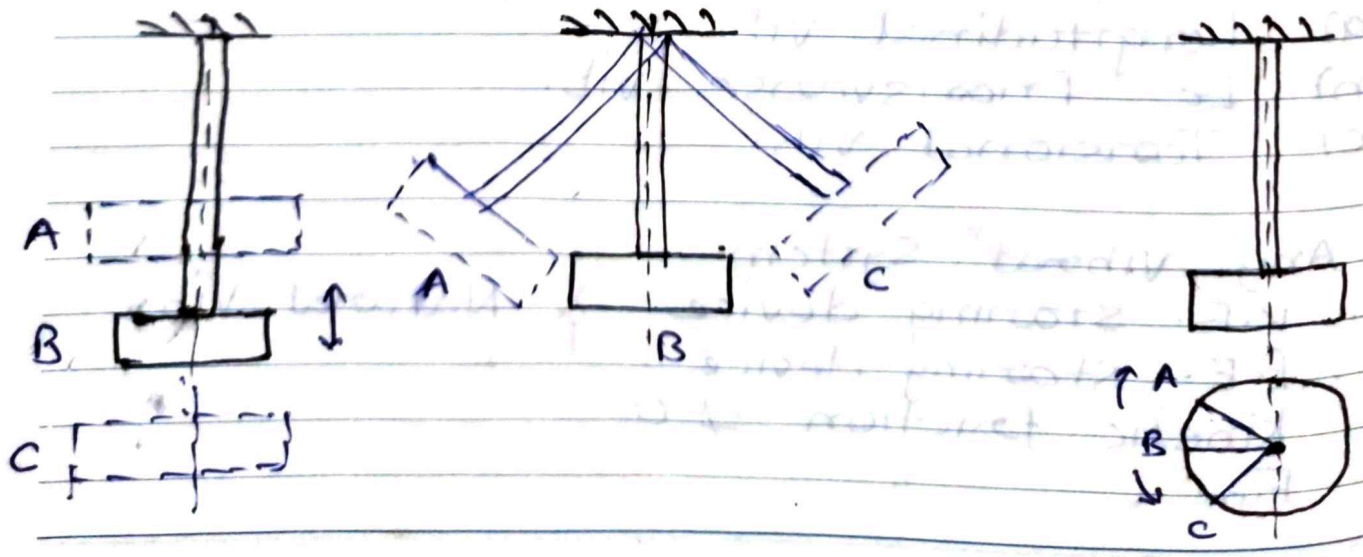
2. Based on stress developed -
(a) Longitudinal vib.
(b) ~~to~~ Transverse vib.
(c) Torsional vib.

Any vibrat^n system
1. K.E. storing device } Natural Vib.
2. P.E. storing device
3. Kinetic friction $\neq 0$
4. Fun

1. Longitudinal vibration - When the particles of shaft moves along the axis of the shaft to & fro in the longitudinal direction, then the vibration are known as long. vibration.

2. Transverse vibration - When the particles of the shaft move perpendicular to the axis of the shaft on either side in the transverse direction, then the vibrations are known as transverse vibration.

3. Torsional vibration - When the particles of the shaft move in angular or circular form, i.e. alternately twisting & ~~de~~ untwisting the shaft about its axis are known as torsional vibration.



4. Free or natural vibration - When a system vibrates without any external force, then the vibration is called free vibration.

5. Damped vibration - When resistance is acting to overcome friction in order to reduce vibration. In this system the vibration die down. There will be reduction in amplitude for every cycle of vibration.

6. Forced vibration - When any system vibrates due to effect of external forces, the vibration is called forced vibration.

7. Damped vibration -

1. Over damping - If $\xi > 1$ ⇒ Coulomb's damping
(Door) ⇓
No vibration

2. Critical damping - If $\xi = 1$ ⇒ viscous damping
(gun) ⇓ ⇓
No vibrat^n (much fast than O.D)

3. Under damping - If $\xi < 1$ ⇒

Vibration with freq. $\omega_d = \sqrt{1-\xi^2}\,\omega_n$
= Const

where $\xi = \frac{c}{2\sqrt{sm}}$ = Damping factor/ratio

c = coeff. of damping ; m = mass of system
s = stiffness

8. Resonance - When the frequency of the external force is equal to ~~freq.~~ natural freq., resonance takes place.

i.e. Amplitude or deformation will reach to max^m at resonance & the system will fail due to breakdown.

9. Frequency - Freq. is the no. of times the motion repeats itself per unit time. (Hz) = cycle/s

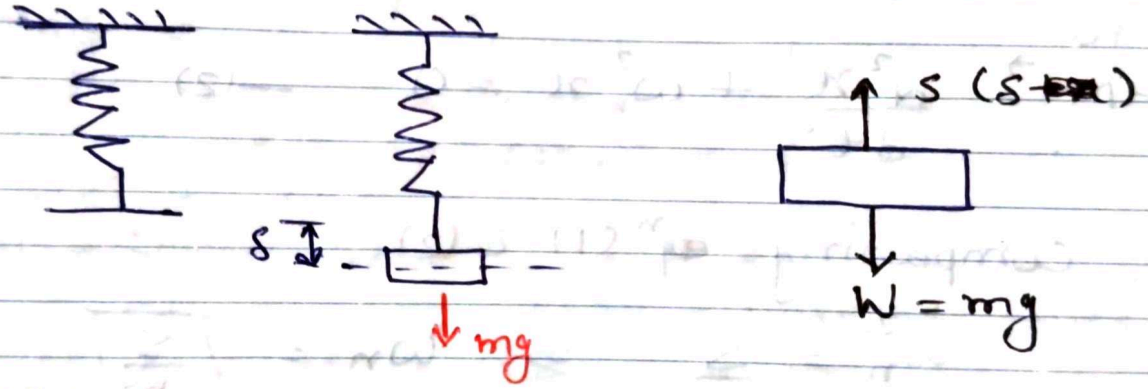
- The vibration in which there is no friction at all, as well as there is no unbalanced force after the initial release of the system are known as N.V.

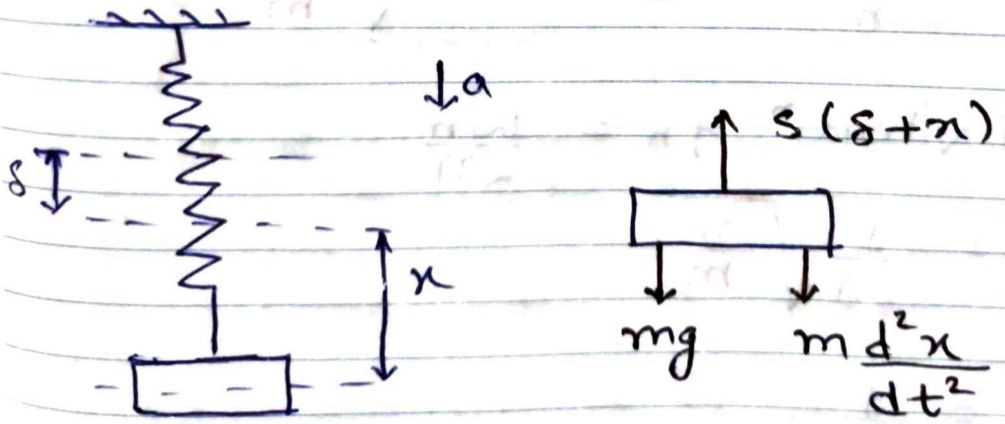
Natural freq. of free long. vibrat^n

1. Equilibrium method -

Let s = Stiffness of element
m = mass of element
δ = static deflection

consider a spring whose one end is fixed & other end is free.
if A mass m is attached to the spring, then spring gets elongated by δ.





spring force is equal to gravitational force.

i.e $mg = s \cdot \delta$

the eq^n of motion of mass m after a time interval t second is

$$m\frac{d^2x}{dt^2} = -sx$$

$$m\frac{d^2x}{dt^2} + sx = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m}x = 0 \quad -(1)$$

A/c to SHM $\Rightarrow$ $$\frac{d^2x}{dt^2} + \omega_n^2 x = 0 \quad -(2)$$

Comparing eq^n (1) & (2)

$$\omega_n^2 = \frac{s}{m} \Rightarrow \omega_n = \sqrt{\frac{s}{m}}$$

& N. freq of vibrat^n $f_n = \frac{\omega_n}{2\pi}$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

Since $\frac{s}{m} = \frac{g}{\delta}$

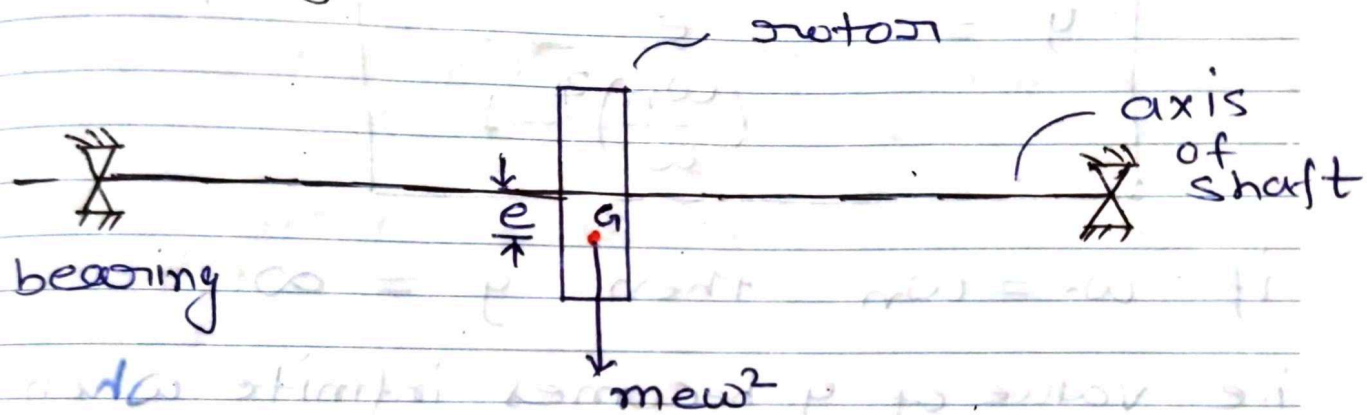
$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

10. Forced damped vibration - Complete reality.

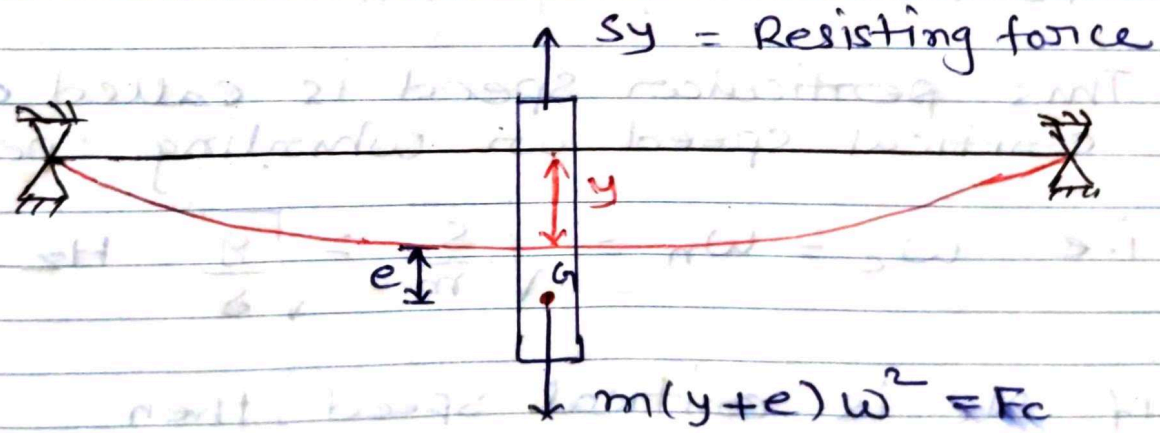
When kinetic friction $\neq 0$

& Fun $\neq 0$

11. Whirling of shaft -



After some time



$$m(y+e)\omega^2 = Sy$$

$$my\omega^2 + me\omega^2 = Sy$$

$$me\omega^2 = (S - m\omega^2)y$$

$$m e \omega^2 = y \cdot m \omega^2 \left(\frac{s}{m\omega^2} - 1 \right)$$

$$e = y \left(\frac{\omega_n^2}{\omega^2} - 1 \right)$$

$$\boxed{y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}}$$

if $\omega = \omega_n$ then $y = \infty$

i.e value of y becomes infinite when circular freq. ω becomes equal to natural freq. ω_n.

This particular speed is called as critical speed or whirling speed.

i.e $\omega_c = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\delta}}$ Hz

if N_c is critical speed, then

$$2\pi N_c = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\delta}}$$

$$N_c = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \text{ rps.}$$

The shaft at which the shaft runs so that the additional ~~speed~~ deflection of the shaft from the axis of rotat^n becomes infinite is known as whirling.

- Degree of freedom -

The min^m no. of independent intensive property / variables required to define the position or motion of the system is known as DOF of the system.